Documentos de Trabajo 21

Introducing an analysis of fertility recuperation and its first empirical findings about European's fertility.

Ruben Castro

udp
UNIVERSIDAD DIEGO PORTALES
facultad de economía y empresa

Introducing an analysis of fertility recuperation and its first empirical findings about Europeans' fertility.

Ruben Castro
November 2010

Abstract. The concept of fertility recuperation is based on the association between birth timing and completed fertility. Fertility recuperation focuses on the interplay between a defined benchmark, most commonly the age at first birth, and the fertility behaviors after the benchmark is reached. Within qualitative analysis of this association, however, whether recuperation took place is not clear nor is the mechanism of recuperation if it occurs. To determine if and how recuperation takes place, in this article I propose a quantitative framework. The cohort's completed fertility is decomposed as a linear combination of the average age at first birth(A), average fertility per year between the first child and the maximum age for fertility (B), and proportion childless (C). The key point is that at the heart of fertility recuperation lies the idea of comparing cohorts in terms of the *speed* at which subsequent births occur after the first child, of which B is a quantitative summary measure. The association of *A* with *C*, on the other hand, is shown to better be explored separately because it is affected by different selection issues. An empirical application is conducted then. The analysis of survey data from West European countries reveals that recuperation is found not only among post-transitional cohorts (born after 1950), but also among pre-transitional cohorts (born between 1931-1946). The pre-transitional cohort compensates lower A's with lower B's and the post-transitional cohort compensates higher A's with higher B's. Regarding the size of recuperation, the compensating changes on B across cohorts are substantially higher than that necessary to counteract the changes on A. Both selection issues and changes in the "completed fertility conditional on age at first child" explain this result. The compensation patterns are also visible in Eastern European countries, though the patterns for post-transitional cohorts are unclear, likely due to the economic changes in the early 90s.

Keywords: fertility recuperation, measures of fertility, low fertility countries.

Introduction

During the last few decades, many developed countries have shown later ages of entry into motherhood and lower fertility rates, which has triggered a discussion about the subjacent relationship between those statistics (Kohler et al 2002). Several papers have argued about the influence of a wide range of factors that explain such associations, most notably the continuous advance of women into the labor market, accompanied by higher education and a change in life styles (see Frejka et all 2008 and Kohler et al 2002 for a detailed discussion). Ages at entry into motherhood in several developed countries have continued rising to the present time[1], perhaps enhanced by high youth unemployment rates and high housing prices (Adsera 2005), and are likely to keep rising in the future (Lesthaeghe and Willems 2004, Frejka and Sardon 2004). The extent to which ages at childbearing, commonly regarded as a barometer for the overall situation of fertility, might also predict low levels of completed fertility is a priority topic among researchers and policy makers.

At the same time, a milestone of this demographic evidence emerged first among Nordic countries, where no strong negative relationship between early in-life fertility indicators (as the average age at

[1] See Nations Economic Commission for Europe (UNECE), "The Statistical Yearbook of the Economic Commission for Europe 2003".

first birth) and completed fertility was observed in the 1950s and 1960s birth cohorts (Kohler et al 2001, Frejka and Calot 2001). This absent or tenuous association between early life fertility and completed fertility was coined, broadly, "fertility recuperation" (one early article is Lesthaeghe and Willems 1999), and different pathways that could explain the occurrence of fertility recuperation are: public policies (Rønsen and Skrede 2008 discuss the case of Nordic countries), the accommodation of family and carriers (a very insightful discussion and empirical evidence is shown in Dorantes and Kimmel 2005), and "stopping behaviors", i.e., the idea that most women attain their ultimate fertility shortly after their first birth (Knodel 1987, also see a recent analysis on Van Bavel 2004).

Under the idea of stopping behaviors, in particular, the concept of fertility recuperation may seem irrelevant, since complete fertility is not going to be affected at all by postponements of the first birth; however, i) birth spacing has been spaced more than 6 years apart, on average, for around 15% of mothers in the US (Anderton et al 1997), ii) a substantial group of mothers do enter motherhood at a late age, thus facing a binding maximum age of fertility: between 20% and 40% of mothers born in different European countries around 1960 entered motherhood after 33 years of age, as shown in Rendall et al 2010, iii) even in the presence of stopping behaviors, women still choose how many births to have, which might be associated with their age at first child, and iv) populations characterized by late motherhood could be substantially restricted by a maximum age at childbearing. All these reasons indicate an association between age at first birth and completed fertility. And even in the case of "pure" stopping behavior, an index of recuperation should reflect that exact recuperation takes place.

But population-level assessments of fertility recuperation provided by current demographic methodology are mostly qualitative. Without a precise definition of recuperation it is not clear if any recuperation took place. Furthermore, in the context of the current small changes in the mean age at first birth (A), a qualitative analysis might fail to identify patterns in the association between A and cohort's complete fertility.

Consider a comparison of two consecutive cohorts where A rose from 35 to 38, while completed fertility dropped from 1.9 to 1.8; did any post-first-birth behavior change between the cohorts?. Also, consider a change in A from 23 to 24 associated with a decrease in fertility of 0.1, versus a change in A from 33 to 34 associated with a decrease in completed fertility of 0.1; are those cases equal in terms of fertility recuperation? It is much more difficult to compensate for the one year delay in the second case. A precise measure is necessary to answer those questions. Another qualitative methodology is based on the comparison of age-specific fertility rates between two cohorts. This kind of analysis shows how a drop in early fertility can be counteracted by late fertility, in accordance with the previously cited evidence where increases in A appeared to not necessarily be associated with lower fertility. While this analysis incorporates a more precise mechanism of recuperation, apparently it does not integrate a quantitative measure of postponement and recuperation.

At the heart of the concept of fertility recuperation is the speed at which new births arrive in a cohort. In this sense, fertility recuperation integrates both fertility rates and time. In this article a summary measure of "fertility rates *after* the occurrence of the first birth (B)" is proposed. An index of fertility recuperation, clearly associated with changes in fertility after the first birth is also proposed, with the aim of enhancing the quantitative explorations of fertility recuperation within demographic research. The occurrence of the first birth is a natural choice to measure postponement, while the birth rate after the first birth is a measure of the "speed" at which new births arrive in a cohort; these measures have their own meaning as demographic concepts and are articulated together within the idea of fertility recuperation (which can be defined in terms of those variables, as in McDonald 2002 for example). A quantitative demographic index of recuperation can be defined as the observed change in B as a proportion of the necessary change to exactly counteract the postponement. The interpretation of B as a behavioral variable goes along the same lines as completed fertility conditional on A.

Because *A* and *B* are observed only among mothers, the proposed analysis applies mostly to comparing cohorts of mothers, which introduces selection issues into the interpretation of the analysis. Aggregate-level methodologies based on cohort's completed fertility have the same problem, because A is observed only among mothers (see Section 1.1). Separately exploring mother's fertility and childlessness would be clearer because childlessness and mother's recuperation are substantially different social processes (see Bavel and Kok 2010 discussion of childlessness in the early XX century birth cohorts) and because economies of scale in the raising of additional children, although these economies have not been firmly confirmed (Folbre 2008). In consequence, perhaps more is gained than lost by focusing either on postponement and childlessness as an issue, and postponement and recuperation among mothers as another issue.

Section 1 discusses the conceptuality and Section 2 conducts empirical applications based on survey data from European Countries; last section concludes.

Section 1: Concepts

1.1 A demographic analysis of fertility recuperation among mothers and childless women

The study of aggregated indicators using population level data remains an important branch of demographic research. It requires data of reasonable availability in most countries, and its conclusions are easily readable from the perspective of the whole country's demographic outcomes, which perhaps explains why this kind of analysis provides the "big picture" that more often than not is used to portray demographic concepts across time and places. This article is focused on this kind of analysis.

The concept of fertility recuperation refers to the idea that changes in the timing may or may not be associated with changes in the cohort's completed fertility. Although the concept of fertility recuperation is intuitively related to postponements of fertility that are counteracted by higher fertility behaviors, the multiplicity of behaviors justify a definition of fertility recuperation as the study of the relationship between birth timing and number of births. Fertility recuperation can be produced by i) changes in fertility preferences, costs of children, labor markets or any other factor that induces a joint re-optimization of both birth timing and quantity of births, and ii) causal effects of timing on quantity. The aggregated indicators of fertility reflect at least those two kinds of recuperation at the same time.

But because measures of postponement (birth timings, and ages at partnering for example) are observed almost exclusively among mothers, the association of a cohort's completed fertility (CCF) and postponement is probably jeopardized. The change in the observed *A* is the outcome of i) women deciding to delay births and ii) women deciding to join or not to join motherhood. So there is a reverse causation from childlessness(C) to A, unless the decision of having children is random with respect to A. In consequence, the observed CCF-A association could appear positive, even though a "true" negative association exists. The C-A association has the same bias. The association of mother's fertility with A, on the other hand, has the advantage that mother's A is observed, though it is probably true that selection into motherhood jeopardizes the comparisons of mother's fertility between two cohorts.

Thus all three associations are subject to some kind of bias. Separately analyzing mother's fertility and A, on one hand, and C-A on the other can simplify the interpretation of results, though. Besides, the analysis of mother's fertility can be complemented with several other variables; in the case of this study, mother's fertility could be A-standardized to distinguish between changes in the distribution of women by A from changes in the values of A-specific subsequent fertility rates. Another advantage of separately analyzing mother's fertility and childlessness is the different inner nature of those processes. The process of deciding to remain childless is probably very different from choosing additional fertility *once* the switch into family life style is already made. When exploring specific behaviors, most studies focus either on childlessness or on fertility among mothers as the final outcome. Micro level studies putting together both groups of women under the same perspective do so in order to better understand mother's behavior, and the selection correction is done without clearly exogenous variables or instruments (with some exceptions, as Rodgers et al 2001's genetic variation across Danish twins).

The case comparing early and late fertility across cohorts is illustrative. While the interplay between those rates certainly defines a more specific mechanism of recuperation, the inclusion of childlessness (C) in the analysis changes both early and late fertility rates, therefore making it hard to analyze the role of *C* in the early-to-late analysis. As is shown on Appendix A.2, the observed relation between early and late fertility rates became a mixture of a) the early-to-late fertility association among mothers only, b) the *C* to mothers' early fertility rate association, c) the *C* to mothers' late fertility association and d) other terms. Two examples are discussed in Section 1.3, showing that adding *C* to the analysis can seriously complicate the interpretation of results.

Current methodologies used to analyze fertility recuperation at the aggregated level are mostly qualitative. This study proposes a quantitative measure of recuperation based on the interplay between postponement and mothers' fertility rate after the first birth, which is easily articulated within the idea of fertility recuperation. An index of recuperation is proposed, based on the duplet of age at first birth (A) and subsequent fertility rate (B), upon which two cohorts are compared; the index is defined as ratio of the change in B to the change in B that would exactly counteract the change in *A*. Additionally, *B* can be decomposed as the product of *A*-conditional *B* rates times the distribution of *A*, which makes further quantitative exploration of *B* possible. Regarding the connection of *B* with individual behaviors, all the micro-level phenomena that are related with completed fertility conditional on *A* maps directly into *B*, since *A*-conditional fertility is just a linear function of *A*-conditional *B*.

The idea behind the analysis through B is as follows. If "Marcela" had her first birth at 31 years old and attained a total of three children, while "Violeta" had her first child at 38 and also attains three children, then how much more intense was Violeta's subsequent fertility rate, on average, in order to attain the same fertility?. If a maximum age for fertility is set at, say, 45, then Marcela had, on average, 2/(45-31)=0.14 additional births per year, while Violeta had, on average, 2/(45-38)=0.28 additional births per year, i.e., twice as much as Marcela, reflecting the fact that she attained the same fertility with only half the available years. Note the nonlinearity of this perspective: a unitary change in *A* represents a 7% or a 14% change on the remaining fertility years if *A* was originally 31 or 38, respectively.

Of course, B is not the only way of summarizing the speed at which new births arrive in a cohort, and even the idea of speed is not the only way of measuring fertility recuperation. B arbitrarily assumes full exposure from the first birth up to a maximum age. Alternative measures have probably alternative assumptions regarding exposure. B is a somewhat crude measure of the "speed" of fertility after the first birth. Another measures, with a higher role for the timing of 2+births for example, depend on the conceptual definitions used. The definition of B in this article follows a simple definition of fertility recuperation and allows a linear decomposition of cohort's complete fertility in terms of A, B and childlessness.

1.2 A summary measure of the subsequent birth rate (*B*), and the cohort's completed fertility expressed as "*A* times *B* times P".

The following exercise rests in the simplifying approximation that mortality is zero before age 45. The cohort completed fertility is therefore defined as the total number of births in a cohort over the total number of persons in the cohort. The fertile life span is assumed between fifteen and forty-five years old. Subscripts *i* are used to index each woman. Women in the cohort are split into mothers and non-mothers, whose total numbers are *M* and *NM* respectively. The cohort completed fertility (CCF) became:

$$\text{CCF} = \frac{\sum_{i=1}^{M} b_i}{M + NM} = \frac{\sum_{i=1}^{M} b_i}{M} \frac{M}{M + NM} \qquad (1)$$

Where b_i stands for *i*-th women's total number of births. Therefore, as shown in Equation 1, the cohort completed fertility can be expressed as mothers' completed fertility rate times the proportion of mothers in the cohort. Next, births are split up into first births (b^1) and superior-births (b^{2+}). By definition, the total number of first births equals the number of mothers. b^{2+}, in the other hand, can be replaced by the product of "b^{2+} per year" times "number of available years", where years refer to the years between *A* and the age of 45:

$$\sum_{i=1}^{M} b_i = \sum_{i=1}^{M} b_i^1 + \sum_{i=1}^{M} b_i^{2+} = M + \sum_{i=1}^{M} (45 - A_i) \frac{b_i^{2+}}{(45 - A_i)} \qquad (2)$$

The total number of years available to have 2+ births, 45-A_i, can be thought of as "exposure". The $b^{2+}/(45\text{-}A_i)$ ratio, on the other hand, will be given an interpretation at an aggregated level, later, and it is noted by B_i. By replacing Equation 2 with Equation 1, and reorganizing, CCF can be expressed as:

$$CCF = Mother's\ CCF \cdot C = (1 + a \cdot B + \text{cov}(a_i, B_i)) \cdot P \qquad (3)$$

where a_i is just 45-A_i, a is the average of a_i, B is the average of B_i and P is the proportion of women who had children. Therefore mothers' complete fertility happens to be a very simple function of the average exposure (a) times the average lifetime additional births per year (B), plus the covariance between those terms (the "1" in the formula reflects the fact that all mothers have a first child). If this covariance is zero, or very low, then mothers' complete fertility can be understood as the simple product of two means, a and B. The empirical analysis below shows that this covariance is indeed very low in nearly all cohorts analyzed.

It is important to note that *B* brings no new data into the analysis, but new statistics about the same data. The same individual-level data used to compute total cohort fertility and average age at first birth can be used to compute *B*. Also note that *B* can be computed without access to micro data; *B* can be derived from data on *A*-conditional completed fertility and the distribution of women by *A*, because women of the same *A* have by construction the same exposure and therefore the average ratio equals the ratio of the averages.

At the aggregate level, *B* is a summary measure of birth rate between the first birth and a certain maximum age. As is proven in Appendix A.1, *B* is the weighted average of *A*-conditional parity-conditional birth rates, where the weights are first the distribution of women by A and second the A-specific proportion of person years spent at each parity.

1.3 Important examples

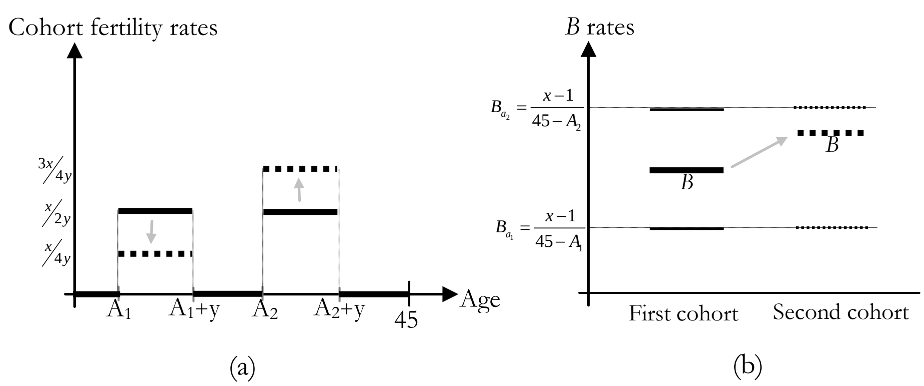
These examples help the exposition of ideas and discuss important issues behind the idea of fertility recuperation.

The stopping behavior

In a nutshell, if (observed, average) birth intervals are not too long, and remain unchanged after entry into motherhood is delayed, then the completed fertility might be barely changed. In other words, if women are going to attain their completed fertility within, say, 7 years after the first birth, then entering motherhood at 25 or 30 will probably not make a big difference in terms of completed fertility.

Consider first the highly stylized case of a cohort where half of the women have a first birth at A_1 and the other half at A_2, though all women have exactly x births in the following y years after the first birth. Without loss of generality, for simplicity assume that births are distributed such that the A_1-women's birth rate between A_1 and A_1+y remains constant, and assume the same for A_2-women. During their birth years, the birth rate for both groups of women is x/y.

Figure 1. Stylized example of postponing in the context of stopping behavior.



The continuous line on panel (a) of Figure 1 shows the cohort's birth rate by age. In the ages between A_1 and A_1+y, the cohort's fertility rate is half of x/y, because half of the women (the A_2-women) are not having any births. For similar reasons, the cohorts' fertility rate is also half of x/y in the A_2 to A_2+y interval. In any other age, the birth rate is zero.

Assume then in the next cohort everything is equal, except that 75%, instead of 50% of women decide for A_2. Then the fertility rates became as shown in the dashed line in Figure 1. Note that A-conditional birth rates remain constant. But the cohort's early birth rate decreased while the late birth rate increased. The comparison of cohorts indicates that exact recuperation took place, as the decrease in early fertility was exactly counteracted by late fertility.

The B rate also went up. The subsequent fertility rate of A_1-women, noted B_{A1} on panel (b) of Figure 1, equals (x-1)/(45-A_1) on both cohorts, while B_{A2} equals (x-1)/(45-A_2) on both cohorts. B_{A2} is bigger than B_{A1}, because A_2-women attained the same completed fertility in less time[2]. Therefore B grows from 50%B_{A1}+50%B_{A2} in the first cohort to 25%B_{A1}+75%B_{A2} in the second. The change in B is also of exact recuperation (which is visible using the index of recuperation, defined in the next Section).

The distinction of recuperation with or without changes in B_A is relevant for the understanding of recuperation. Note that in this example no change in B_A took place, only the distribution of A changed, though both analyses (panel *a* and *b* on Figure 1) acknowledged recuperation. The analysis of B rates, however, in addition to providing a quantitative assessment of recuperation, can be decomposed to show that A-conditional behaviors did not change, as is shown in the next Section.

The stopping behavior, nevertheless, does also change B_A rates if the number of births by A_1–women was originally different than that of A_2-women.

The adherence behavior

Adherence behavior happens when changes in A across cohorts are not accompanied by changes in B_A rates. So, for example, the new group of A_2-women "adheres" to the previously observed behavior among woman of A_2. This kind of phenomenon is observed, in the empirical section of this study, among cohorts in East European countries.

Modify the stopping behavior discussed in the previous example by assuming that women of A_1 have $x/2$ births instead of x births, and leave everything else equal. As in the previous example, B_A rates were not changed after the postponement took place, but both analyses (Panel *a* and *b* of Figure 1) would show that more than exact recuperation took place. Panel (a) of Figure 1 would should that lost fertility at early ages is more than entirely recovered at late ages, while Panel (b) would show that more than counteracting B took place (which is visible using the index of recuperation, defined in the next Section). Thus there were no changes in A-specific behavior but women who postponed adhered to the existing A-specific patterns, which in this case implied a higher fertility.

Including childlessness

As it is shown algebraically in the next paragraph, the inclusion of childless women in the analysis will increase or decrease the observed recuperation depending not only on the association between postponement and childlessness, but also the triple interaction of childlessness-early-late fertility rates at the same time, which is not clearly integrated into the idea of recuperation.

First, consider the case of stopping behavior of Figure 1. Since the area under the birth rate on Panel (a) represents the completed fertility, the difference in the areas (cohort 2 minus cohort 1) can be used as a measure of recuperation. Panel (a) shows zero difference in the areas, thus there is full recuperation. Now add childlessness to the stopping behavior, assuming that the percentage of

[2] A few words about the rationality for B_{A2} being bigger than B_{A1} are perhaps necessary. The measure of the subsequent fertility rate discussed in this article is intended to provide a summary of the fertility behavior after the first birth, not the trajectories within the first birth and the age of 45. The only aspect of the trajectories that is captured by B is on what is related to the completed fertility. In this example, the trajectories differ on the years spent without births (45-A_1-y years and 45-A_2-y respectively) which matters in terms of the completed fertility.

mothers is P in the first cohort, and $P+\Delta P$ in the second. In the first cohort, the new birth rates correspond to P times what they were. In the second cohort, they correspond to $(P+\Delta P)$ times what they were. Therefore the difference in areas is $y\cdot\Delta P$ (Equation 4). Thus if ΔP is zero then $\Delta areas$ is zero, meaning full recuperation, as it was without including childlessness. If the postponement was associated with a decrease in the percentage of mothers instead, i.e. $\Delta P<0$, then $\Delta areas<0$ as well, meaning that recuperation is less than full. So this goes along the intuitive lines.

$$\Delta areas = y\left[\frac{3}{4}(P+\Delta P)+\frac{1}{4}(P+\Delta P)-\frac{1}{2}P-\frac{1}{2}P\right]=y\cdot\Delta P \qquad (4)$$

In the case of the adherence behavior, however, the same exercise implies the following $\Delta areas$:

$$\Delta area = y\left[\frac{3}{4}(P+\Delta P)+\frac{1}{8}(P+\Delta P)-\frac{1}{2}P-\frac{1}{4}P\right]=\frac{y}{8}\cdot[P+7\Delta P] \qquad (5)$$

thus if ΔP is zero then $\Delta areas>0$ meaning that recuperation is more than full, even though the postponement was not associated with changes in childlessness. If the postponement was associated with a decrease in the percentage of mothers instead, i.e. $\Delta P<0$, then $\Delta areas$ could be bigger, equal or smaller than zero. If P is 80% and ΔP is -10%, for example, the inclusion of childlessness would make recuperation more than full, even tough the postponement actually increased childlessness.

1.4 Applications

Different applications of the B rate are discussed in this Section, both to illustrate and to explore the analysis of recuperation.

The recuperation index

An index of recuperation can be defined based on the comparison of two cohorts in terms of their average age at first birth and their average subsequent fertility rate. One cohort is used as reference and the index is computed as the observed change in B, B_2-B_1, over the change in B that would leave the completed fertility equal between the cohorts, B_{er}-B_1, where B_{er} is that of exact recuperation[3]. The index of recuperation (IR) is therefore defined as:

$$IR=\frac{B_2-B_1}{B_{er}-B_1} \qquad (6)$$

and its possible values are:

- Negative, if the observed changes in B reinforce rather than counteract the changes in A.
- Zero, if no change in B took place.
- Between zero and one, if changes in B were in the right direction but not big enough.
- One, if exact recuperation took place.
- Bigger than one, if the recuperation did more than counteract the change in A.
- Undefined, if no change in A was observed in the first place.

[3] The calculus of B_{er} is done assuming that $\text{cov}(A_i,B_i)$ remains constant.

The index can be computed from survey data or from aggregate data, as is shown in the next application.

The index in terms of cohort-level measures of fertility

The index of recuperation can be approximated using existing measures of fertility. In the worst case, where all the researcher has is the cohort completed fertility (*CCF*), the percentage of mothers (P) and the average age at first birth, then Equation 3 can be used to compute *B*, assuming that the covariance shown Equation 3 is low enough to be approximated to zero[4]. *B* = [(*CCF-Cov)*/*P*-1]/*A*. Note: In the empirical Section, the covariance is actually found to be low in all but one of the cohorts analyzed.

The index of recuperation can, therefore, be expressed along the same lines. If the mother's completed fertility (*CCF*P*, noted "F") changes from F_0 to F_1, and *A* changes from A_0 changes to A_1, then the necessary *B* to exactly counteract the change is the (F_0-1)/A_1. Therefore the index of recuperation, namely the observed change in *B* as a percentage of the necessary change in *B* is:

$$IR = \frac{\frac{F_1 - 1}{A_1} - \frac{F_0 - 1}{A_0}}{\frac{F_0 - 1}{A_1} - \frac{F_0 - 1}{A_0}} = 1 + \frac{\Delta F / (F_0 - 1)}{\Delta A / (45 - A_0)} \qquad (7)$$

the index of recuperation, in this case, is simply the ratio of the change in fertility (relative to the original additional fertility) to the change in age at first birth (relative to the original time available to have 2+ births). In the case that fertility remained constant across cohorts, ΔF is zero and the index is one, meaning that *B* exactly counteracted the change in *A*. In the case that *A* decreased by 3% (relative to the original time available) and *F* increased 3% (relative to the original additional fertility), then the index is zero, meaning that *B* did not change at all. In the case that the changes in A and F are 3% and 1% respectively, the index is 0.66, meaning that the change in *B* (a negative change in this case) was not enough to counteract the longer exposure period.

Decomposing fertility recuperation

Differences in a demographic outcome can be decomposed using different methodologies, as for example Das Gupta (1993). Differences in fertility across time or cohorts, along these lines, could be decomposed on A-components and B-components. Changes on *B* itself, being simply the weighted average of *A*-specific *B* rates (B_A), as shown in Appendix A.1, could be decomposed into changes on B_A and changes on the distribution of women by A.

Adding structure to B_A

The analysis proposed in this study is facilitated by assuming that the B_A rates follow a pattern with respect to *A*. Probably the easiest one is a linear relationship:

[4] If a constant covariance is added to Equation 7, then IR=1+[ΔF/(F_0-1-cov)] / [ΔA/(45-A_0)]. Thus, if cov is low in relation to F-1 it could be taken out without a substantial loss of precision.

$$B_A = \beta_0 + \beta_1 \cdot a \qquad (8)$$

where a=45-A, meaning the exposed time to have additional births. Under this specification, B simplifies to a linear function of two parameters and the average exposed time: $B= \beta_0+\beta_1\bar{a}$. Also under this specification, changes in the distribution of A are fully represented by changes in $\bar{A}$, while changes in B_A are represented by changes in β_0 and β_1. The mothers' completed fertility simplifies to $1+\beta_0\bar{a} +\beta_1\bar{a}^2+\beta_1\sigma_A$, where $\beta_1\sigma_A$ is the covariance between A_i and B_i; comparing $\beta_1\bar{a}^2$ with $\beta_1\sigma_A$, for example, it is reasonable to expect the covariance to have low influence, because σ_A is typically around 3, while $\bar{a}^2$ is 289 if the mean age at first birth is 28.

Section 2: Empirical application

In this Section the analysis of recuperation discussed so far is applied to data from European countries, with a focus on the major trends across time.

2.1 Data

Survey data is used to conduct all the analysis in this section. All the data is publicly available through their respective survey websites. Cohorts of women in Europe are analyzed using data from the third wave of the European Social Survey (ESS), conducted in 2006[5], and the first wave of the Generation and Gender Survey (GGS), conducted mostly in 2005[6]. Because of substantial socioeconomic differences, ex-soviet countries are labeled as East European countries, and non ex-soviet countries are labeled West European countries (see a comparison of West and East Germany in Kreyenfel 2003). Cohorts are grouped in five-year intervals. Sample sizes by countries are shown in Table 3.1 on Appendix A.3. While actuarial data shows that around 3% of women alive at 15 do die before age 45, the accuracy of the approximation implicit in B (see section 1.2) is lower among the earlier cohorts analyzed in this article, which are born in the early XX century. All countries included to date in the Max Plank Institute of Demographic Research (MPIDR)'s Human Fertility Database are also used to confirm some of the results using population-level data.

During the time span of this data (1929 to 1963 birth cohorts), cohort's complete fertility dropped from 2.4 to 1.9 on WE countries, while remained around 2.0 on EE countries (see Table A.31 in Appendix A.3). Although all components of Equation 3 showed substantial changes (see a decomposition of changes in cohort's completed fertility on Figure A.31), the comparison of A and B across time reveals a positive relation between those variables (Figure A.32), in concordance with the idea of fertility recuperation as defined in this article; this figure also shows a fundamental difference between WE and EE countries.

2.2 The covariance between A_i and B_i

Cohort's completed fertility in Equation 3 was depicted as a function of two averages (A and B) and the covariance of A_i and B_i. Figure 2 shows the covariance as a percentage of the mothers' fertility.

[5] More information available at http://www.europeansocialsurvey.org.
[6] More information available at http://www.unece.org/pau/ggp.

On average across cohorts, the covariance is mostly negative, meaning that higher ages at first birth are correlated with lower subsequent birth rates:

Figure 2. Cov(Ai,Bi) by country (average across cohorts).

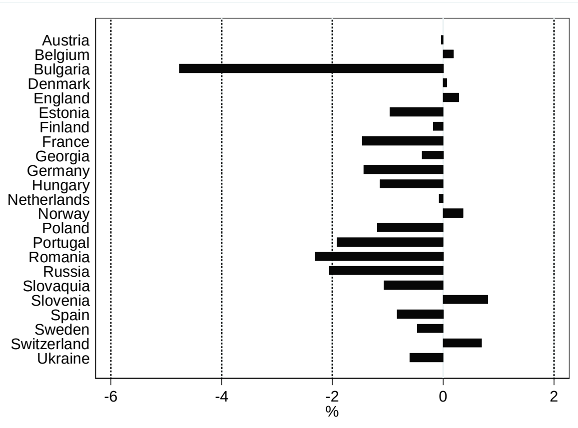


Figure 2 shows that, on average across cohorts, the ratio of covariance versus mothers' completed fertility ratio is low. The ratio lies between a +/- 2% band around zero, with the exception of Bulgaria's 4.7%. The average across countries is only -0.8%. It is reasonable, then, to describe mothers' completed fertility in terms of *A* and *B* only, which makes the analysis simpler. Along these lines, the changes in the covariance across consecutive cohorts are also probably too low to influence the results; the analysis of recuperation, done below by comparing consecutive cohorts, is done by simply assuming that the covariance remains constant between consecutive cohorts.

Note that *B* is a function of the association between A_i and B_i, because *B* can be decomposed as a function of *A*-conditional *B*, times the distribution of *A* (see appendix A.1). It is the covariance term, as it appears on Equation 3 that can probably be taken out of the analysis for the sake of simplicity, but the association of A_i and B_i still influence the value of *B*.

2.3 Different kinds of recuperation across time

Although the index of recuperation can be computed from the comparison of any two cohorts, in some cases the concept of recuperation itself might not be relevant, for example in the case of cohorts where a profound fertility transition takes place. Indeed, birth cohorts between 1944 and 1953 can be called "transitional" because they show the biggest drops in completed fertility. The following table classifies the consecutive cohorts across countries according to their recuperation behavior.

Recuperation is classified as forward if increases on *A* were at least partially counteracted, and backward if decreases in *A* were at least partially counteracted. *B*-recuperation notes mothers' recuperation through *B*, and childless-recuperation means cohort's recuperation through changes in the percentage childless.

Table 1. Number of countries where consecutive cohorts show fertility recuperation, by direction and type of recuperation, type of country and birth cohort.

Birth cohorts	Recuperation's direction WE-countries			Recuperation's direction EE-countries		
	Forw.	No rec.	Backw.	Forw.	No rec.	Backw.
Recuperation through *B*:						
1931-1936	10	4		6	4	
1936-1941	11	3		6	3	1
1941-1946	4	9	1	6	3	1
1946-1951	2	7	5	2	6	2
1951-1956	2	3	9	3	7	
1956-1961	1	3	10	3	5	2
Recuperation through Childlessness:						
1931-1936	3	9	2		9	1
1936-1941	7	6	1	3	5	2
1941-1946	3	6	5	2	6	2
1946-1951	3	7	4	3	5	2
1951-1956		8	6	5	4	1
1956-1961		8	6	4	1	5

*Author's calculation based on survey data from ESS and GSS.

Birth cohorts born around 1941 to around 1951 are the ones that experienced the biggest changes in completed fertility and are those where recuperation behavior appears less commonly. Table 1 also shows that recuperation through *B* is not a recent behavior; both WE and EE countries commonly experienced "recuperation backwards" among pre-transitional cohorts (birth cohorts from 1931 up to around 1941), and "recuperation forward" appears common on post-transitional cohorts (birth cohorts from around 1951 up to 1961). This pattern is especially clear in WE countries. Population-levels computation of the *B* rates, done by applying Equation 7 to country data available on MPIDR's Human Mortality Database (Czech Republic, Estonia, Netherlands, Russia, Slovakia and Sweden) also shows this pattern.

Table 1 also classifies the number of countries by cohort according to their recuperation through changes in the percentage childless. Half the cases show "no recuperation", in concordance with the idea that increases (decreases) in the average age at first child are accompanied with increases (decreases) in the percentage childless. However, the other half of the cases did show recuperation through childlessness; although this may seem a random result, EE countries shows recuperation down among pre-transitional cohorts and recuperation up in post-transitional ones. Nevertheless, WE countries do not show this pattern.

2.4 The magnitude of recuperation

In the cases where recuperation was observed, either down or up, as depicted in Table 1, the index of recuperation was computed for each country. In nearly all cases, the IR was well above 1, meaning that increases (decreases) in *B* were more than enough to counteract the increases (decreases) in the average age at first child. The percentual increases in *B* rank between 10 and 20%, in absolute value, but this is well above the increase needed for an IR=1.

Figure 2. Average IR and % change in *B* by type of country and % change in *A*.
(Only consecutive cohorts were *B*-recuperation is observed are included. Values of IR are top-coded at 5)

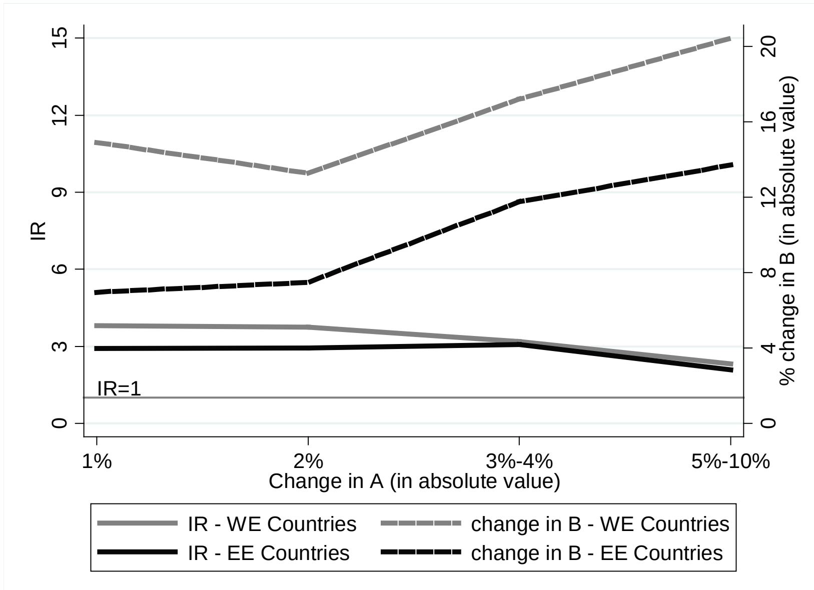


Figure 2 shows both changes in *B* and the index of recuperation, among consecutive cohorts where *B*-recuperation was observed. Changes in *B* do not represent a precise quantification of recuperation; IR shows that the changes in *B* rank between 3 and 13: too high in terms of the necessary change to exactly counteract the change in *A*. Nearly all values of IR by country (not shown) are higher than 1. This result could be anticipated without the IR itself, since exact counteraction requires more or less similar changes in *A* and *B*, which is not the case in Figure 2, where changes in *B* are substantially higher than changes in *A*.

Population-levels computation of the B rates, done by applying Equation 7 to country data available on MPIDR's Human Mortality Database (Czech Republic, Estonia, Netherlands, Russia, Slovakia and Sweden) also shows B rates higher than 1. This finding implies that, when recuperation is found, mothers' fertility is higher (lower) after an increase (decrease) in the age at first child. If childlessness is added into the analysis, the IR can change drastically, because childlessness might either counteract or reinforce the change in *A*. Table 1 shows in fact that in half the cases there is childlessness-recuperation.

Figure 2 shows that when recuperation is observed, bigger changes in the average *A* are, on average, accompanied by bigger changes in *B*. Those bigger changes in *B* are, however, not bigger in terms of their impact for mothers' fertility; Figure 2 shows that bigger changes in *A* are associated with equal changes in IR.

2.5 Changes in *B*: due to changes in A-specific behaviors or to changes in A?.

Changes in *B* are the outcome of changes in *A*-specific *B* rates, noted B_A rates, and changes in the distribution of *A*. Changes in *B* can be decomposed into those terms by using Das Gupta (1993)'s decomposition method. To obtain more stable results, B_A is assumed a linear function of *A*, as shown in Equation 8. The decomposition of *B* is performed on each country and consecutive cohort.

ΔB_A note the change on B induced by changes in A, and ΔB_{BA} note the change on B induced by changes in the B_A rates. The ratio of the former to the latter is shown in Table 2.

Table 2. Ratio of ΔB_A to ΔB_{BA} , and changes in A by type of country and birth cohort.

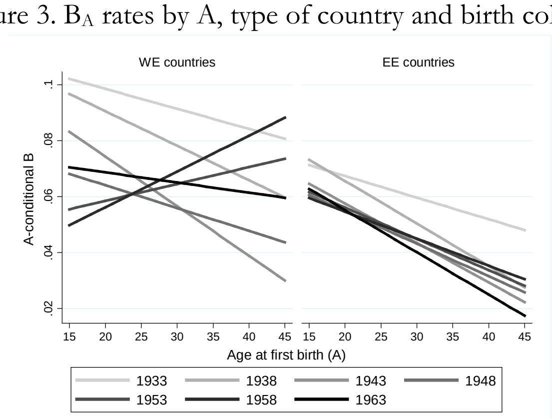
Birth cohorts	Median Ratio of ΔB_A to ΔB_{BA}		Average change in A	
	WE Countries	EE Countries	WE Countries	EE Countries
1933-1938	-0.08	-0.09	-4.6	-3.4
1938-1943	-0.07	-0.13	-3.5	-2.1
1943-1948	-0.03	-0.15	-0.4	-1.4
1948-1953	0.01	-0.04	1.4	-0.2
1953-1958	0.10	-0.12	4.3	-3.8
1958-1963	0.01	-0.07	3.0	-0.0

*Only consecutive cohorts were B-recuperation is observed are included. The median, rather the mean ratio is displayed because small denominators make some ratios attain very extreme values.

The median ratio shows that most of the changes in B are actually explained by changes in B_A, not by changes in A, especially in the case of WE countries. This result holds on all consecutive cohorts, either where high or low changes on A took place (see last columns of Table 2). The clear higher importance of B_A rates highlights that understanding the recuperation behavior across time of women of the same age at first is a key piece of knowledge about fertility recuperation.

The next figure shows the evolution of B_A rates across cohorts. A profound distinctive pattern between type of country emerges. WE countries experienced big changes in B_A rates across cohorts, while EE countries are much more stable. WE countries evidence first a proportional decrease in B_A rates, and later a change in the slope, which became very low or even positive in recent cohorts, meaning that late mothers' B_A rates are comparable with that of women of young motherhood. The last cohorts in this study show that B_A rates of women of young motherhood are very similar between WE and EE countries, on average, but late mothers have a much higher B_A in WE countries

Figure 3. B_A rates by A, type of country and birth cohort.



However, throughout this paper there is no evidence that recuperation behavior of EE countries is significantly lower, although late mothers in that type of country have much smaller B_A rates than early mothers. The reason has to do with the last two columns of Table 2. EE countries have been experiencing negative changes in A, i.e., younger ages at first birth, therefore women are concentrating in the relatively higher B_A rates.

Section 3: Conclusion

The demographic idea of recuperation is broadly defined based upon the comparison of two cohorts and refers to the association of timing of births and complete fertility. It includes re-optimizations of time and quantity of children and causations from the former to the later. Although originally stated as recuperation from late motherhood, it is conceptually and empirically an issue of recuperation from early motherhood too. Fertility recuperation, nevertheless, emphasizes the idea that later ages at first child may be associated with lower fertility.

Regarding an aggregated-level analysis of recuperation, a key problem is that measures of postponements are observed mostly among mothers. Comparing consecutive cohorts in terms of mother's fertility versus postponement, or childlessness versus postponement, or cohort's fertility versus, is jeopardized by this issue. Focusing only on mothers or only on childless women can facilitate further exploration of the interpretation of results, though. In addition, mother's fertility and childlessness are processes of fairly different nature.

Current methodologies do not quantitatively articulate postponement and subsequent recuperation. Within qualitative analysis of this association, however, the mechanism of recuperation is not clear, and whether any recuperation took place is not clear either. In this study a methodology for a quantitative assessment of fertility recuperation is proposed. The speed at which new births arrive in a cohort is at the heart of fertility recuperation; along these lines, the proposed analysis is based on a summary measure of the fertility rate per year after the first birth (noted the B rate), which is shown to be the outcome of the distribution of women by age at first birth and the proportions of person years in each parity afterwards. A simple linear combination of i) mean age at first birth (A) , ii) the B rate and iii) proportion childless is shown to closely approximate the cohort's complete fertility, which facilitates the introduction and index of fertility recuperation based on the comparison of two consecutive cohorts and defined as the observed change in *B* divided the change that would exactly counteract the change in *A*. Under some probable assumptions, cohort-level data can be used to compute B and the index of recuperation.

The interpretation of B is that of a summary measure, the result of several different factors, which can be grouped in selection, changes in fertility conditional on A, and changes on A. Although B is a somewhat crude measure of the "speed" of fertility after the first birth, the cohort's completed fertility can be viewed as a simple linear combination of B and A and childlessness; besides, B itself arises from the simple definition of fertility recuperation used here.

Simple empirical applications of this framework are used to analyze some major trends in fertility recuperation. Data comes from survey data on European countries (General Social Survey and Gender and Generation Survey) It is found that early XX century birth cohorts showed as much "backward *B*-recuperation" (compensating for younger *A*'s) as cohorts born around 1951-1961 showed "forward *B*-recuperation" (compensating for later *A*'s). Unexpectedly, most *B*-recuperation appears well above the necessary levels, which is confirmed using cohort-level data from the Max Plank Institute of Demographic Research (MPIDR)'s Human Fertility Database. A puzzling result is that West European countries also show the pattern of backwards and forward recuperation in terms of compensating the changes in *A* by changes in childlessness.

The idea of stopping behaviors states that most women attain their complete fertility shortly after the first child, so changes in A shouldn't be associated with changes in complete fertility, which implies an index of recuperation of 100%. However, the index of recuperation is well above this level, at

300%, meaning an over-compensation for changes in A. It is not clear at this point whether selection issues, stopping behaviors or other factors explain this result.

The main difference found between West European and East European countries is that while young mothers show similar *B* rates in each type of country, on average, the latter has much smaller *B* rates among late mothers; this West-East difference has not arisen in terms of lower *B*-recuperation because *A* in those countries is decreasing (which is also observed, as a general trend, in the Human Mortality Database).

A general lesson of this study is the importance of studying *A*-specific fertility behaviors. Those behaviors are at the heart of differences across countries and time, and the inherent selection issues involved in the analysis are perhaps easier to tackle than those of more general fertility measures. And finally, future research on the index of recuperation could acknowledge the drop in fecundability after age 35 (see for example Rizzi et. Al. 2005), which would emphasize the changes in post-first-birth behaviors necessary to compensate for postponement.

References

[1.] Adsera, A. "Vanishing children: From high unemployment to low fertility in developed countries". 2005. *American Economic Review Papers and Proceedings*, 95, 189-193.

[2.] Amuedo-Dorantes, C., & Kimmel, J. "The motherhood wage gap for women in the United States: The importance of college and fertility delay". 2005. *Review of Economics of the Household*, 3, 17–48.

[3.] Anderton, D. L., Barrett, R. E., & Bogue, D. J. 1997. The population of the United States. New York: Free Press.

[4.] Billari, F. C., Kohler, H.-P., Andersson, G. & Lundström, H. "Approaching the limit: Long-term trends in late and very late fertility". 2007. *Population and Development Review*, 33:149–70.

[5.] Das Gupta P. Current Population Reports. U.S. Bureau of the Census; Washington, DC: 1993. "Standardization and Decomposition of Rates: A User's Manual" Series P–23, No 186.

[6.] Folbre, N. 2008. Valuing children: Rethinking the economics of the family. Cambridge, MA: Harvard University Press.

[7.] Frejka, T. and G. Calot. "Cohort reproductive patterns in the Nordic countries". 2001. *Demographic Research* 5(5): 125−186.

[8.] Frejka, T. and Tomas Sobotka and Jan M. Hoem and Laurent Toulemon. "Summary and general conclusions: Childbearing Trends and Policies in Europe". 2008. *Demographic Research*, Max Planck Institute for Demographic Research, Rostock, Germany, vol. 19(2), pages 5-14, July.

[9.] Frejka T. and J.-P. Sardon. Childbearing Trends and Prospects in Low-Fertility countries: A Cohort Analysis. 2004. Dordrecht, New York and London: Kluwer Academic Publishers.

[10.] Hans-Peter Kohler and Francesco C. Billari and José Antonio Ortega. "The Emergence of Lowest-Low Fertility in Europe During the 1990s,". 2002. *Population and Development Review*, The Population Council, Inc., vol. 28(4), pages 641-680.

[11.] Knodel, John. "Starting, Stopping, and Spacing During the Early Stages of Fertility Transition: The Experience of German Village Populations in the 18th and 19th Centuries". 1987. *Demography* 24:143–62.

[12.] Kreyenfeld M. "Crisis or adaptation—reconsidered: a comparison of East and West German fertility patterns in the first six years after the 'Wende"'. 2003. *Eur J Popul.*; 19:303–29.

[13.] Lesthaeghe, R. and P. Willems. "Is low fertility a temporary phenomenon in the European Union". 1999. *Population and Development Review* 25(2): 211–228.

[14.] McDonald, Peter. "Low fertility: unifying the theory and the demography." Australian National University, Mimeo, 2002.

[15.] Nations Economic Commission for Europe (UNECE). "The Statistical Yearbook of the Economic Commission for Europe 2003". Available at www.unece.org.

[16.] Rendall M, Aracil E, Bagavos C, Couet C, Derose A, Digiulio P, Lappegard T, Robert-Bobée I, Rønsen M, Smallwood S, Verropoulou G. "Increasingly heterogeneous ages at first birth by education in Southern European and Anglo-American family-policy regimes: A seven-country comparison by birth cohort." 2010. *Popul Stud* (Camb);64(3):209-27.

[17.] Rizzi, E., A. Rosina, and B. Colombo. "Age effect: results from a detailed prospective study on daily fecundability." 2005. *Revue d'épidémiologie et de santé* publique /Epidemiology and Public Health, Special Issue on "Late parenthood: risks of failure to conceive, for pregnancy outcome and for the child" 53-HS2: 57-63.

[18.] Rodgers, Joseph L., Hans-Peter Kohler, Kirsten Ohm Kyvik, and Kaare Christensen, "Behavior Genetic Modeling of Human Fertility: Findings From a Contemporary Danish Twin Study". 2001. *Demography* - Volume 38, Number 1, pages 29-42.

[19.] Rønsen, Marit. "Fertility and family policy in Norway -A reflection on trends and possible connections". 2004. *Demographic Research*, Volume 10, Article 10, Pages 265-286.

[20.] Rønsen M. and K. Skrede. "Fertility trends and differentials in the Nordic countries – Footprint of welfare policies and challenges on the road ahead". 2008. *Vienna Yearbook of Population Research*. 103-123.

[21.] Van Bavel, Jan and Kok, Jan. "Pioneers of the Modern Lifestyle? Childless Couples in the Early Twentieth-Century Netherlands". 2010. *Social Science History*. vol:34 issue:1 pages:47-72.

[22.] Van Bavel. 2004. Detecting stopping and spacing behaviour in historical demography. A critical review of methods. Population-E 59(1):117-128.

A.1 Decomposing *B*

In this article B_i is introduced as an individual level variable. In this appendix it is proved that averages of B_i are interpretable in term of births hazard rates.

All mothers in a cohort can be classified by their age at first birth, A, their age at second birth, A_2, if any, and so on. Assume that mothers do not have more than one birth in the same year, and that all mothers expose to birth hazard-rates until a maximum age of, say, 45. Equation 1.1 shows that B can be expressed as the weighted average of A-specific B rates:

$$B = \frac{1}{N}\sum_{i=1}^{N} B_i = \frac{1}{N}\sum_{A=15}^{45}\left(\sum_{i_A=1}^{N_a} B_{i_A}\right) = \sum_{A=15}^{45}\frac{N_a}{N}\frac{\left(\sum_{i_A=1}^{N_a} B_{i_A}\right)}{N_A} = \sum_{A=15}^{45} d_A \cdot B_A \quad (1.1)$$

where i indexes each mother and i_A indexes mothers of A age at first birth, N is the number of mothers, N_A is the number of mothers of A age at first birth, B_A represents the average B among mothers of A age at first birth and d_A represents the proportion of mothers with A age at first birth.

The next paragraphs show that B_A is the weighted average of the parity-specific hazard rates, always among women of A age at first birth, where the weights are the proportion of person years exposed on each parity. Equation 1.2 expresses B_A as the summation of each additional birth ($b^{n\text{-}th}$ notates the *n-th* birth):

$$B_A = \frac{1}{N_A}\sum_{i_A=1}^{N_A} B_{i_A} = \frac{1}{N_A}\sum_{i_A=1}^{N_A}\frac{b_{i_A}^2 + b_{i_A}^3 + \cdots}{45 - A} \quad (1.2)$$

The hazard rate of second births, noted HR^2_A, is the average ratio of the number of second births to the person years exposed to the risk of a second birth, which are noted PY^2_A. Therefore the number of second births is the product of the hazard rate times the exposed time. Equation 1.3 express B_A as a weighted average of parity-specific hazard rates:

$$B_A = \frac{1}{N_A}\left(\frac{HR_A^2 \cdot PY_A^2}{45 - A} + \frac{HR_A^3 \cdot PY_A^3}{45 - A} + \cdots\right) = \frac{HR_A^2 \cdot PY_A^2}{N_A(45 - A)} + \frac{HR_A^3 \cdot PY_A^3}{N_A(45 - A)} + \cdots = \sum_{b=2}^{\max b}\frac{PY_A^b}{PY_A} HR_A^b \quad (1.3)$$

As a final note, for future research it may be useful to show that A-specific parity-specific hazard rates can be expressed as functions of A-specific parity-specific path-specific values of B. By splitting the summation of second births onto a series of summations of A_2-specific second births, Equation 1.4 shows that HR^2_A can be expressed as the weighted average of the $B^2_{A,A2}$ (the average B rate of second births among mothers of A and A_2 age at first and second birth respectively), and the weights are the proportion of PY^2_A that were exposed by mothers of each (A,A_2):

$$HR_a^2 = \frac{\sum_{i_a=1}^{N_a} b_{i_a}^2}{PY_a^2} = \frac{\sum_{a_2=a+1}^{45}\left(\sum_{i_{a,a_2}=1}^{N_{a,a_2}} b_{i_{a,a_2}}^2\right)}{PY_a^2} = \frac{\sum_{a_2=a+1}^{45}\left((a_2-a)\frac{N_{a,a_2}}{N_{a,a_2}}\sum_{i_{a,a_2}=1}^{N_{a,a_2}}\frac{b_{i_{a,a_2}}^2}{(a_2-a)}\right)}{PY_a^2} = \frac{\sum_{a_2=a+1}^{45}\left((a_2-a)\frac{N_{a,a_2}}{N_{a,a_2}}\sum_{i_{a,a_2}=1}^{N_{a,a_2}} B_{i_{a,a_2}}^2\right)}{PY_a^2} =$$

$$= \frac{\sum_{a_2=a+1}^{45}\left((a_2-a)N_{a,a_2}B_{a,a_2}^2\right)}{PY_a^2} = \frac{\sum_{a_2=a+1}^{45}\left(PY_{a,a_2}^2 B_{a,a_2}^2\right)}{PY_a^2} = \sum_{a_2=a+1}^{45}\left(\frac{PY_{a,a_2}^2}{PY_a^2}B_{a,a_2}^2\right) \quad (1.4)$$

The hazard rate of third births follows alone the same line, but with the additional complication that for each *a* there is an array of a_2 and for each a_2 there is an array of a_3. So in this case the process is similar than that of second births, but with a double process of obtaining weighted averages. First, averaging over all different values of a_2, and then over all the different values of *a*:

$$HR_a^3 = \sum_{a_2=a+1}^{45}\left(\frac{PY_{a,a_2}^3}{PY_a^3}\sum_{a_3=a_2+1}^{45}\left(\frac{PY_{a,a_2,a_2}^3}{PY_{a,a_2}^3}B_{a,a_2,a3}^3\right)\right) = \sum_{a_2=a+1}^{45}\left(\frac{PY_{a,a_2}^3}{PY_a^3}B_{a,a_2}^3\right) \quad (1.5)$$

where the triad a,a_2,a_3 refer to women of *a*, a_2 and a_3 age at first, second and third birth, respectively. Hazard rates of superior orders follow along the same lines.

A.2 Childlessness and the demographic comparison of cohort's early and late fertility.

Postponing could be associated with childlessness, if repeated postponing increases the likelihood of childlessness. This association is not easily seen in the comparison of early-in-life fertility with late-in-life fertility rates, because the fertility rate affects both the early and the late fertility rate. Consider the following examples. To simplify the discussion, assume no mortality before age 45. The fertility rate of mothers at age z is noted F_z and it is computed as the ratio of total births at age z (noted b_z) over total number of mothers. If F_z is multiplied by *P*, the percentage of the cohort who had children, the cohort fertility rate at age z is obtained, noted CF_z. The total size of the cohort is *N*. Then:

1) The ratio of mothers' early to late fertility of mothers is equal to the ratio of cohort early to late fertility. The proof is straightforward; the numerator and denominator of the first ratio are multiplied by *P* to obtain the second ratio. Another perspective to the same issue is that the inclusion of childless women decreases fertility rates by an amount proportional to the rates themselves:

$$F_z - CF_z = \frac{b_z}{N \cdot P} - \frac{b_z}{N} = \frac{b_z}{NP}(1-P) \quad (2.1)$$

2) In a Cross-Section or longitudinal analysis, childlessness almost surely introduces variation across cohorts. If data from different countries (or different births cohorts) are used to regress CF_{40} on CM_{20}, the OLS coefficient is a weighted average of several different components. To simplify notation, use y_c and x_c for F_{40} and F_{20} respectively, where x_c and y_c are already measured in deviations from the mean across countries. The OLS coefficients from regressing either mothers' or women's fertility rates are:

$$\beta^M = \frac{\sum_{c=1}^{C} x_c \cdot y_c}{\sum_{c=1}^{C} x_c^2} \quad ; \beta^W = \frac{\sum_{c=1}^{C} (x_c + f_{x_c,p_c})(y_c + f_{y_c,p_c})}{\sum_{c=1}^{C} (x_c + f_{x_c,p_c})^2}; \quad f_{x_c,p_c} = \frac{P_c X + P_c x_c - S_{x,p}}{P} \quad (2.2)$$

where P and X are the mean values of P_c and x_c respectively, and $S_{x,p}$ is the covariance between x_c and P_c. In Equation 2.2 is difficult to interpret β^W; however, if the summation of three-some products is approximated to zero, β^W became:

$$\beta^W \approx \left[\beta^M + \frac{\beta_{p,x}}{K_y} + \frac{\beta_{p,y}}{K_x} + \frac{\sigma_p^2}{\sigma_x^2} \right] \frac{\sigma_x^2}{\sum (x_c + f_{x_c,p_c})^2} \quad (2.3)$$

where $\beta_{p,x}$ is the coefficient from regressing p on x, and $\beta_{p,y}$ is the coefficient from regressing p on y, and K_x is $(X\text{-}S_{x,p})/P$. In other words β^W conveys information about the slopes of y versus x, p versus x and p versus y, plus other terms of cumbersome interpretation. It is hard, therefore, to interpret the weighted mixture of these (possibly counteracting) coefficients. In the case that cohorts have almost the same level of childlessness, including childlessness in the analysis does not change the OLS coefficient, *regardless* of P (because $f_{xc,pc}= f_{yc,pc}=0$). In the case that most of the variation comes from P_c, then the OLS coefficient became the ratio of average late fertility to average early fertility, *regardless* of the variation in P_c (because $f_{xc,xp}=P_cX/P$ and $f_{yc,pc}=P_cY/P$).

Finally, the previous discussion applies to regressing not fertility rates, but the change-across-time on fertility rates instead.

A.3 Tables and Figures

Table A.31. Sample sizes by Country.

Country	Observations	Source	Country	Observations	Source
Bulgaria	2,923	GGS	Estonia	528	ESS
France	3,156	GGS	Finland	609	ESS
Georgia	3,131	GGS	Ireland	500	ESS
Germany	3,054	GGS	Norway	484	ESS
Hungary	4,561	GGS	Poland	492	ESS
Netherlands	2,537	GGS	Portugal	890	ESS
Romania	3,851	GGS	Slovaquia	440	ESS
Russia	4,255	GGS	Slovenia	498	ESS
Austria	696	ESS	Spain	537	ESS
Belgium	539	ESS	Sweden	578	ESS
Denmark	486	ESS	Switzerland	609	ESS
England	803	ESS	Ukraine	787	ESS

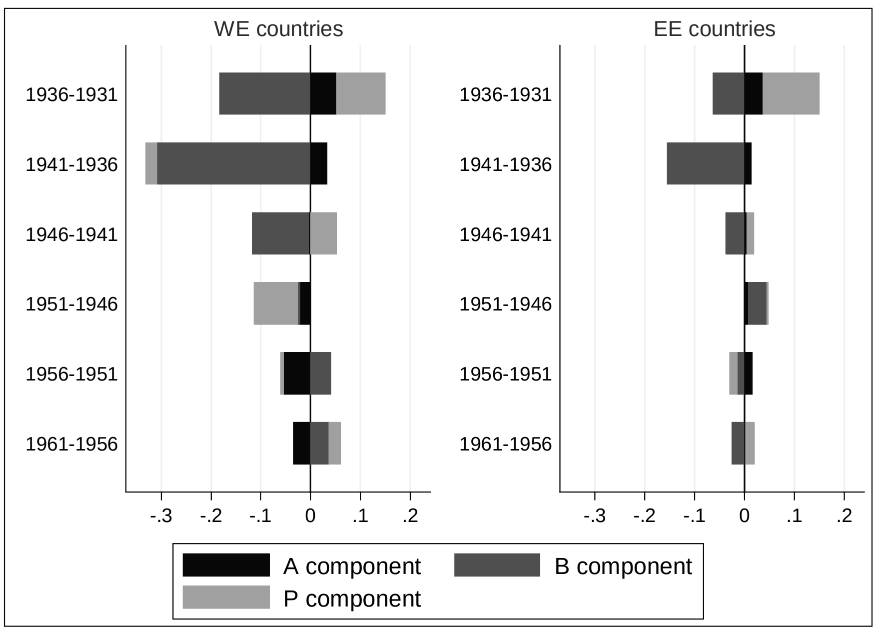
Source: Author's calculations based on survey data. Observations correspond to women that were included on this article, not to the overall size of the surveys.

Table A.32. Cohort's completed fertility by birth cohort

Cohort*	Average of WE Countries	Average of EE Countries
1931	2.44	2.04
1936	2.42	2.11
1941	2.12	1.97
1946	2.04	1.95
1951	1.90	2.00
1956	1.86	1.98
1961	1.92	1.99

*Source: Author's calculations based on survey data. Five-years cohorts are labeled by the central year.

Figure A.31. Decomposition of changes on cohort's completed fertility across time.
(Average of WE and EE European countries is shown)



* Source: Author's calculations based on survey data. 1936-1931 stands for the comparison of the five cohorts born around 1936 with the five cohorts born around 1936. Methodology for the decomposition is taken from Das Gupta (1993).

Figure A.32. Values of A and B across cohorts.