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Holding Company Discounts and Business Groups Optimal Bailout of Subsidiaries

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Abstract

In this paper, we develop a simple two period model of the holding company-subsidary relationship that analyzes the option faced by controlling shareholders to optimally bailout subsidiary companies using the holding company's financial resources. This behavior is rational under several incentive structures originated in the presence of group externalities, synergies and private benefits of control. The results of our model are consistent with recent empirical evidence of holding company discounts, and with the existence of diffuse limits among the affiliated firms of a group. Our results allow us to interpret the presence of holding company discounts as evidence that bailing-out may arise for reasons other, and less conflictive, than the perpetuation of tunneling benefits by the controlling shareholder, such as the perpetuation of benefits of reputation and group synergies.

Keywords: Corporate Governance, Corporate Finance, Holding Discount, pyramidal structure

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I. Introduction

A business group is a collection of separate firms tied by ownership relations and which, in spite of their legal autonomy, tend to respond to a common “group” strategy (Lefort (2010)). As it turns out, most firms in emerging economies and several developed economies are linked in some way or another to a business group that exercises tight control over the firms and owns a large fraction of their shares (Colpan et al (2010), La Porta, Lopez-de-Silanes and Shleifer (1999)).

As a consequence, a growing body of literature in corporate governance and corporate strategy has shifted its focus away from the standard agency problem between managers and dispersed shareholders, and has, instead, looked closely into the relationship between minority and majority shareholders. In particular, it has been argued that business groups are prone to carrying inefficient investment and generating minority shareholder expropriation, especially when control is exercised through complex mechanisms such as pyramid schemes, cross-holdings, and dual-class shares. In those cases, the agency problem is exacerbated because, on the one hand, ownership concentration insulates the controller from the market for corporate control, and on the other, control is executed by a shareholder that holds a relatively small fraction of the cash flow rights (Lefort and Walker (2007), Bebchuck (1999), Bebchuk, Cohen, and Ferrell (2004) and Wolfenzon (1999)).

Interestingly, many of these studies recognize that one of the most salient characteristics of business groups, especially in emerging economies, is that they are persistent in time and able to adapt to most changing situations. The large amount of evidence compiled in Colpan et al. (2010) shows that business groups have been able to grow and increase their scope and self-intermediation practices even during times of fierce economic reform and deregulation. This kind of evidence has supported a more favorable view of business groups, arguing that economic groups are a natural and efficient way for firms to deal with imperfect capital markets, poor institutions, corruption, and other imperfections that frequently plague emerging economies. In this context, economic groups arise in order to fill (or to take advantage of) the voids left by poor institutions. Hence, they could represent an advantage in dealing with a corrupt government, a highly regulated economy, and a poor judiciary system (Khanna and Palepu (1997)).

The combined control of several companies may also create operational synergies, which might be related to economies of scale and scope in product and factor markets, due to poor basic services like electricity, postal service, or others. In particular, internal capital markets, that is, the headquarters' collection and allocation of funds to the different companies in a group could create value in a credit constrained world (Stein (1997)). Other financial synergies may arise because of the possibility for business groups to liquidate assets of specific firms in response to a general downturn (Shleifer and Vishny 1997), and because of risk diversification that might be valuable to investors in economies with underdeveloped capital markets.

An interesting consequence of the existence of synergies and private benefits of controlling different companies through a business group is the creation of incentives that can trigger an opportunistic behavior consisting in bailing out distressed companies controlled by the group with financial resources belonging to other companies of the same business group. In the academic literature, this behavior has been labeled as propping or inverse tunneling (Riyanto & Toolsema 2008). Naturally, in order for this behavior to be optimal, the initial costs of bailing out should be expected to be recovered in excess by the business group in the future. Future benefits may arise either from the possibility of keeping tunneling benefits (Friedman et al. (2003)), or from the possibility of benefiting from synergies obtained by the group that otherwise would be lost.

A common way to organize a business group is through a pyramid (Almeida and Wolfenzon (2006), Bertrand (2000), Colpan et al. (2010)). Usually, in the top of such structures, a holding company, controlled by the business group's controlling shareholder, owns shares of the group's different subsidiary companies, which, in turn, own shares of other firms in the group. Complicated pyramidal structures may have several of these layers of group companies. It has been argued that pyramids arise in order to allow business groups to achieve control of large assets with small amounts of capital. The rationale for such a behavior is the possibility of tunneling private benefits through the pyramid.

Recently, Almeida and Wolfenzon (2006) have argued that this explanation is at odds with

evidence showing that many business groups organized as pyramids have very minor separation between cash flow rights and voting rights (Lefort and Walker (2000)). Almeida and Wolfenzon's explanation for the existence of pyramids is that business groups choose "...the pyramidal structure because of the payoff and financing advantages it provides when new firms are expected to yield low-security benefits relative to the required investments."[†] Almeida and Wolfenzon's explanation is interesting, among other things, because it seemingly turns around the standard vision regarding the existence of pyramids. Common wisdom associates pyramids to a tool implemented to extract private benefits. However, the results of Almeida and Wolfenzon's model indicate that pyramids are the best way to finance the creation of new companies that yield relatively low-security benefits.

Both views are, in fact, consistent, since Almeida and Wolfenzon argue that the pyramid provides a financial advantage to fund new acquisitions in which security benefits are low relative to private benefits. Hence, the controlling shareholder is willing to invest other people's money to gain access to private benefits.

In this paper, we want to contribute to the literature on business groups by adding an overlooked empirical regularity to the complex issues that theoretical models on pyramid structures attempt to explain. Recently, several authors have drawn attention to the existence of a holding company discount in many business groups. Such a discount means that the holding company's market capitalization is less than the market capitalization or economic value of the investments it holds minus the holding company's debt (Net Asset Value or NAV). Rommens, Deloof and Jegers (2004) show that holding companies, which play an important role in corporate finance in Belgium and in other Continental European countries, often trade at a discount to their estimated net asset value (NAV). Lefort and González (2010) and Gálvez (2009) find that in the Chilean case, even holding companies in which all assets are shares in traded companies and thus net asset values are calculated straightforwardly, trade at a 30% gross discount over NAV[‡].

[†] The Journal of Finance, Vol.LXI, No.6, Dec 2006, pp 2640

[‡] For a summary exposition see appendix 1

Holding company discounts are normally attributed to limited free float of the holding company, tax inefficiencies associated with the holding company, and the additional administrative costs any holding company incurs. Rommens et al. hypothesize that, in addition, holding company discounts could be due to the extraction of private benefits by controlling shareholder at the expense of minority shareholders, and to the fact that the estimated NAV may be an overestimation of the actual value of the holding company's investments. However, as Lefort and González (2010) and Galvez (2009) show, discounts are still present when net asset values can be directly obtained from market values.

However, what makes holding company discounts of special interest is that they seem to be counterintuitive in the presence of tunneling. To illustrate this, consider the simple case where a holding company's sole assets are shares of a subsidiary. When private benefits of the controlling shareholder are extracted from the subsidiary to the holding company, these benefits increase the holding company's security benefits, hence security value, and for the same reason these benefits will be shared by the controlling shareholder with minority shareholders at the holding company. Nevertheless, when private benefits are extracted from the subsidiary directly to controlling shareholder's pockets, private benefits are not reflected in Holding Company security value and thus are not shared. It is easy to show that the former case implies a Holding Company Premium, and the later, and consistent with empirical evidence, implies a holding discount.

Since, in general, the controlling shareholder fraction of cash flow rights will be higher in the holding company than in the subsidiary, unless the later is a wholly owned company, incentives will be structured so as to eventually tunnel private benefits from the subsidiary to the holding company, and not the other way around.

Consistent with this intuitive result, Almeida and Wolfenzon's (2006) model predicts that business groups will find it optimal to structure pyramids in order to profit from their financing advantage only in situations where the model parameters predict the presence of holding company premiums, calculated in the way that both ourselves as well as the security markets analysts have

done[§].

Additionally, Almeida and Wolfenson's (2006) model predicts that an unexpected acquisition of a new company into the pyramidal structure will cause a loss of value in the holding company, because this implies additional private benefits to the controlling shareholder, at the expense of the holding company's minority shareholders, which finance part of the acquisition. Our model predicts the same results, reinterpreting Investment as bailout: an unexpected investment by the holding company in an already "tunneled" subsidiary produces a loss for the minority shareholders of the holding company.

In this paper, we develop a simple two period model of the holding company-subsidary relationship that analyzes the option faced by the controlling shareholder to optimally bailout the subsidiary using the holding company's financial resources. This behavior is rational under several incentive structures originated in the presence of group externalities, synergies and private benefits of control^{**}. The general intuition is that when the bankruptcy of a financially distressed subsidiary may cause a loss of benefits to the controlling shareholder, it may be optimal for him to pay a cost in order to bailout the distressed company. This cost will be shared with minority shareholder at the holding company, and will eventually show the existence of a holding company discount, in case this behavior is already internalized in market values, or, alternatively, it will produce a loss of value for the holding company, if such behavior is unexpected.

Moreover, we could advance the idea that the reasons argued by Almeida & Wolfenson (2006) for the existence of pyramidal groups should be extended to include other reasons, mainly, the ability to deploy strategic assets and exploit synergies in new companies^{††}. Almeida & Wolfenson's, and this paper's views are parallel in the sense that pyramids are useful devices to

[§] The Journal of Finance, Vol.LXI, No.6, Dec 2006, pp 2654

^{**} We can interpret the bailout of a distressed subsidiary as the intention to maintain wholistic group benefits such as reputation and synergies, in the same line of reasoning as in Morck, Randall, The Riddle of the Great Pyramids, in: Asli M. Colpan, Takashi Hikino and James R. Lincoln Editors (2010) The Oxford Handbook of Business Groups, Oxford University Press Canada, pp 602-628

^{††} This reasoning belongs to the modern resource based view of the firm, and can be traced back to Edith Penrose's classic work "The theory of the growth of the firm, 1959"

finance new ventures, as well as for protecting existing ones, creating an economic network between the companies of the group^{‡‡}. Consistent with the above, strong incentives may exist to preserve the structure of a group in the presence of financial distress in one of the subsidiaries, in spite of the costs that controlling and minority shareholders must bear.

The results of our model are consistent with empirical evidence of holding company discounts, and they allow us to interpret the presence of such discounts as evidence that bailing-out may arise for reasons other, and less conflictive, than the perpetuation of tunneling benefits by the controlling shareholder, such as the perpetuation of benefits of reputation and group synergies. In other words, the picture that emerges from this model, the holding company discount evidence, and casual evidence of holding companies effectively funding subsidiary companies when they face financial distress, is one of diffuse limits between the subsidiary companies of a group. This representation of a business group is consistent with the numerous descriptions of specific cases occurring in a variety of countries depicted in Colpan et al (2010).

In particular, a conclusion of this paper is that legal boundaries to firms in the context of a business group are less meaningful than expected. Business groups not only may eventually tunnel private benefits from the firms they control or allocate strategic resources among them, but they appear to be willing to forfeit the benefits associated with limited liability rights in the firms they control. The result is a benefit accrued by minority shareholders on downstream companies of the group, partially financed by minority shareholders of upstream companies. When the expected resources at stake in this intercompany resource allocation surpass the expected value of tunneling or other benefits from subsidiaries, the result is the appearance of a holding company discount.

Our model predicts that, when the likelihood of propping is positively related to the business group's intention to perpetuate reputation externalities, a holding discount should be observed. Interestingly, and opposed to the previous result, when the likelihood of propping is positively

^{‡‡} See also Morck, Randall, The Riddle of the Great Pyramids, in: Asli M. Colpan, Takashi Hikino and James R. Lincoln Editors (2010) The Oxford Handbook of Business Groups, Oxford University Press Canada, pp 602-628

related to the business group intention to perpetuate synergies or tunneling, holding company shares will eventually be traded at a premium instead of a discount, when compared to the corresponding NAVs.

The rest of the paper is structured as follows: Section II presents a model of optimal bailout of subsidiaries. We calculate holding company discounts for a number of cases, depending on the nature of the benefits that the controlling shareholder can optimally derive from the pyramid, ranging from externalities to tunneling, to synergies. The decision to bail-out the distressed subsidiary and the welfare implications to minority shareholders given the existence of potential conflicts of interest are also analyzed in this section. Section III addresses empirical implications of the model, and advances some testable hypothesis. Section IV discusses some venues for future research and provides conclusions.

II. A model of optimal bailout of subsidiaries

A. A simple model

We model a business group as two related but legally separate entities: an unlevered holding company (HC), controlled by a controlling shareholder and a partially owned levered subsidiary, controlled by the holding company. The HC's assets are its investments in the subsidiary's shares plus cash (retained earnings). The controlling shareholder holds a fraction h of the holding company's shares, and the holding company holds a fraction f of the subsidiary. We assume $0.5 < h, f < 1$, and that h and f are enough to exercise complete control over the holding company and the subsidiary.

In our model, agents are rational, so they maximize expected cash flows in an educated manner and use all public information; there is a competitive securities market to which shareholders and debt holders have free access. There are neither taxes, information asymmetries nor bankruptcy costs, and discount rates are assumed to be zero.

The subsidiary is a risky productive asset financed with debt and equity. In a two period time frame ($t=0, 1$), the state of nature is revealed in $t=1$ as asset value is achieved by taking one of two possible values: V_u and V_d with probabilities q and $(1-q)$ respectively, where $V_u > V_d$. We denote equity market values as S , we use superscripts H and Aff to denote holding company and the affiliated company respectively. When it is not obvious, we use subscripts 0 and 1 to denote $t=0$ and $t=1$. Face value of debt in $t=1$ is D . We assume $V_u > D > V_d$, so that in the "down or bad" state of nature the subsidiary becomes financially distressed.

Limited Liability (LL) is the legal rule by which, shareholders of a distressed company, have the right but not the obligation to go bankrupt by transferring ownership and control of assets to creditors. For simplicity, we assume that in the case of bankruptcy no negotiations will take place with creditors and no further equity costs will be incurred. Hence, the subsidiary's equity value is equivalent to a call option on the company assets with a strike price equal to the face value of debt.

Conversely, we refer to “rescue” or “bailout”, the case in which shareholders, while having the legal right to go bankrupt, avoid bankruptcy and keep ownership and control of assets by fully repaying debt.

In $t=1$ the state of nature is revealed, and the subsidiary’s asset value is known. If the state is “u” assets are kept or cashed-out at a profit. If the state is “d”, controlling shareholder must decide whether to go bankrupt or to rescue the subsidiary. We assume that cash held initially at the holding company (i.e. retained earnings in the form of liquid assets) equals the amount shareholders must pay to avoid bankruptcy ($Cash=D$). If the controlling shareholder decides to bailout the subsidiary, they will force an equity call at the subsidiary and, assuming minority shareholders do not concur, all new equity will be paid by the holding company from existing cash^{§§}

The following Box summarizes Holding equity valuations or wealth for both types of shareholders and in both states of nature under these assumptions:

Box A

Holding Company shareholders net wealth, in $t=1$, in \$

Base Case: No Benefits

		State of nature “u”	State of nature “d”
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot (f \cdot (V_u - D) + Cash)$	$h \cdot Cash$
	Minority	$(1 - h) \cdot (f \cdot (V_u - D) + Cash)$	$(1 - h) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot (f \cdot (V_u - D) + Cash)$	$h \cdot (f \cdot V_d)$
	Minority	$(1 - h) \cdot (f \cdot (V_u - D) + Cash)$	$(1 - h) \cdot (f \cdot V_d)$

^{§§} By assumption, controlling shareholders have enough voting power to unilaterally approve the equity call. Minority shareholders in the Affiliate will find it optimal not to concur.

Since by assumption $V_d < D$ in the absence of other considerations such as external benefits, synergies, or private benefits of control, there is no economic incentive for the holding company to bailout the subsidiary and give up limited liability. In fact, if the controlling shareholder decided to bailout the subsidiary, he would suffer a change in his wealth of $(1-q) \cdot h \cdot \{f \cdot V_d - Cash\}$, which is a loss given our assumptions. Hence, the expected present value of the holding company assets is $E(S^H) = q \cdot (f \cdot (V_u - D)) + Cash$.

For simplicity we compute holding discounts as the difference between expected present equity value for minority shareholders in the Holding Company (Security Value in Almeida and Wolfenson terminology) and their share in the net asset value (NAV). Similar to market analysts, we calculate NAV as the market value of the holding company's assets, i.e. its share in the subsidiary's equity market value, plus excess cash, minus the holding company's financial debt. We use minority shareholders' valuations as proxies to market value since we assume their shares are the only shares traded in the securities market. For this reason, holding discounts computed in this way implicitly include relative liquidity issues.

In this simple model, the security value of HC's minority shareholders is $E(S^H) = (1-h) \cdot (q \cdot f \cdot (V_u - D) + Cash)$, while their share in NAV is $(1-h) \cdot (q \cdot f \cdot (V_u - D) + Cash)$. Therefore, in this case holding discount is simply zero.

$$(1) \text{ HD}(\text{no benefits, going bankrupt}) = 0$$

It is interesting to see that if, against basic economic sense (as shown above), the controlling shareholder of HC decided to rescue the distressed subsidiary, HC's minority shareholders security

value would be $(1-h) \cdot (q \cdot (f \cdot (V_u - D) + Cash) + (1-q) \cdot f \cdot V_d)^{***}$, while their share in NAV would be $(1-h) \cdot (q \cdot f \cdot (V_u - D) + Cash)$; hence there would be a holding discount, shown as $(1-q) \cdot (1-h) \cdot (f \cdot V_d - Cash) < 0$. Note that this corresponds to the HC minority shareholder's share in the expected net cost of rescuing the distressed subsidiary. In our model this cost is incurred by all the shareholders of the holding company. As we can see, there is no rationale for the controlling shareholder to bailout the subsidiary. Hence the market will assign zero probability of bailout, and security values will reflect that fact. Hence we can write (1) as

$$(1') \quad HD(\text{no benefits}) = 0$$

Intuitively, the controlling shareholder of a business group will only consider rescuing a distressed subsidiary if the benefits of bailing it out are greater than the associated costs. Hence, in the following section we consider variations on our model based on different types of benefits associated to maintaining the pyramidal structure in place by rescuing the distressed subsidiary. We model different characterizations of benefits capture under different assumptions. The inclusion of these benefits changes the incentives structure of the model, allowing for situations where optimal bailouts of the distressed subsidiary arise. The benefits to be considered are External Benefits (characterized by reputation effects) in section B1, Private Benefits of Control (characterized by Tunneling) in section B2, and Synergies in section B3.

B. A Rationale for bailing out subsidiaries

B.1 External Benefits

The model is the same as in section A, but we assume the existence of external benefits X derived from outside the economic boundaries of the business group and accrued directly by all

*** In the case of rescue, as minority shareholders in the Affiliate do not concur to the equity call, the share of the holding company in the Affiliate actually increases from f to some $f < F < 1$, this fact is not used in our models and does not alter our conclusions.

shareholders of the holding company. These benefits may arise from reputation in the business community that produces more favorable customer or supplier relations and new business opportunities.^{†††} External benefits are lost if the subsidiary goes bankrupt.

We assume that X is received by the HC's shareholders in addition to their security value of the subsidiary assets. We divide the external benefit X in two parts: X_1 received by all shareholders of the holding company^{†††} and, X_2 received exclusively by the controlling shareholder. X_1, X_2 are positive and not state dependent. X_1 as well as X_2 are not reflected in the financial statements and public information of the holding company or the subsidiary, and therefore they are not added to security benefits in our definition.

The following Box summarizes wealth values, which include security values and external benefits, for both types of shareholders of the Holding Company, in both states of nature, in $t=1$:

Box B1

Holding Company shareholder net wealth in $t=1$

Case 1: External Benefits of the Business Group

		State of nature "u"	State of nature "d"
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot (X_1 + f \cdot (V_u - D) + Cash) + X_2$	$h \cdot Cash$
	Minority	$(1 - h) \cdot (X_1 + f \cdot (V_u - D) + Cash)$	$(1 - h) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot (X_1 + f \cdot (V_u - D) + Cash) + X_2$	$h \cdot (X_1 + f \cdot V_d) + X_2$
	Minority	$(1 - h) \cdot (X_1 + f \cdot (V_u - D) + Cash)$	$(1 - h) \cdot (X_1 + f \cdot V_d)$

^{†††} For a detailed Survey see Walker, Kent (2010). A Systematic Review of the Corporate Reputation Literature: Definition, Measurement, and Theory. *Corporate reputation Review* 12, 357-387

^{†††} For simplicity we assume benefit X_1 is received in the same share they hold in the holding company.

Bailout decision

Under the previous assumptions and as shown in Box B1, if the state of nature is “u” external benefits will remain in place. Alternatively, if the state of nature is “d” and controlling shareholder lets the subsidiary go bankrupt, all shareholders will lose their share of the external benefits and keep their share of the cash. On the contrary, if the controlling shareholder decides to bail out the distressed subsidiary, shareholders will keep their share of the external benefits by paying the face value of debt using the retained earnings of the holding company.

Since, by assumption, the controlling shareholder at the Holding Company has enough voting power to unilaterally decide the subsidiary’s fate, the decision will be based on the relative wealth of the controlling shareholder under the bad state of nature. According to Box 1, the controlling shareholder will decide to bail out the subsidiary based on the relative importance of his share of external benefits and the net costs of rescue.

If the controlling shareholder decided to rescue the subsidiary, taking as the stand point the non-rescue condition, he would have a change in wealth of $(1-q) \cdot h \cdot \left\{ f \cdot V_d - cash + X_1 + \frac{X_2}{h} \right\}$ which can be negative (loss) or positive (gain) depending on the relative importance of the external benefits compared to the net cost of rescue. Hence the bailout decision will be made if

$$\left\{ f \cdot V_d - cash + X_1 + \frac{X_2}{h} \right\} > 0$$

Therefore, as expected, in the presence of external benefits it is more likely that, in the case of financial distress of the subsidiary, the controlling shareholder will decide to bail it out when, relatively speaking, the external benefits obtained by him are bigger, the level of subsidiary’s debt is lower and the HC’s stake in the subsidiary is larger. Somewhat less intuitive, although unambiguous, is the effect of “*h*”, the controlling shareholder’s stake in the Holding Company. This is because, even though a higher “*h*” increases the external benefits accrued by controlling shareholder, it also increases the net cost of rescue. In other words, the lower the “*h*”, the higher the contribution that minority shareholders will make to the cost of rescue.

Holding Discount

At this point, it should be clear that, in this case, holding discounts will depend, among other things, on the likelihood that the controlling shareholder will decide to bail out the financially distressed subsidiary. In fact, even in the presence of external benefits, if the controlling shareholder decided to let the subsidiary fail (exercise limited liability rights) the observed holding discount in $t=0$ would be zero, as security value and NAV are equal. This is the same result as equation (1).

Conversely, if the controlling shareholder decided to rescue the distressed subsidiary, because

$\left\{ f \cdot V_d - cash + X_1 + \frac{X_2}{h} \right\} > 0$, a holding discount would be observed in $t=0$, given by:

$$(2) \quad HD(\text{external benefits, bailout}) = (1 - q) \cdot (1 - h) \cdot (f \cdot V_d - D) < 0$$

As opposed to Case 0, in the presence of external benefits, the controlling shareholder of a holding company may decide to optimally bail out a financially distressed subsidiary. In this case, a holding discount proportional to the net cost of rescue^{§§§} will be observed if the market expects the subsidiary to be rescued. In turn, the net cost of rescue is less when the stake of the HC in the subsidiary is bigger, and the subsidiary debt is less.

The effects on the shareholders' wealth, and in the holding discount will depend on whether the bailout decision is anticipated by the market or not, in $t=0$.

If the market anticipates a bailout decision, the holding company's security value is discounted in $t=0$, the minority shareholders' initial stock purchases and on-going trades are made at the discounted prices, and no loss of wealth occurs. Notwithstanding the above, when the time comes

^{§§§} Actually, it corresponds to the expected cost of rescue, where expectation is calculated over states of nature

to rescue the subsidiary in $t=1$, minority shareholders may eventually oppose that action. This will be treated in detail when we analyze conflicts of interest.

Now, if the decision to rescue the distressed subsidiary were not internalized by the market in $t=0$ security values, an unexpected bailout would produce an effect on the minority shareholders' wealth, which could be a gain or a loss depending on the parameters of the model. This wealth effect corresponds to, and will be reflected by, the change in the holding discount produced by the announcement of the decision. To see this, let $\Delta(W_m^X)_{nr}^r$ be the change in wealth of the minority shareholders, and ΔHD_{nr}^r be the change in the holding discount level, from bankrupt (no rescue) to bailout (rescue), it can be shown that $\Delta(W_m^X)_{nr}^r = \Delta HD_{nr}^r + (1-q) \cdot (1-h) \cdot X_1$. This means that the change in the holding discount due to an unanticipated bailout underestimates the true change in wealth of the minority shareholders whenever they derive external benefits. If all external benefits are accrued by the controlling shareholder, i.e. $X_1 = 0$ the change in the holding discount level will match exactly the change in the minority shareholders' wealth.

Conflict of interests

It is easy to see that if minority shareholders benefited from the rescue, the controlling shareholder would also benefit. Unfortunately, the reverse is not necessarily true, giving rise to potential conflicts of interest.

Assume that bailing out the subsidiary is the optimum behavior of the controlling shareholder of the holding company. This means that compared to the non-rescue condition, the controlling shareholder will receive a wealth gain $\left\{ f \cdot V_d - cash + X_1 + \frac{X_2}{h} \right\} > 0$ or $h \cdot \{ f \cdot V_d - cash + X_1 \} + X_2 > 0$. The first three terms on the left hand side of this expression correspond to the wealth change accrued by minority shareholders. Hence, minority shareholders may eventually suffer a loss $h \cdot \{ f \cdot V_d - cash + X_1 \} < 0$ while the controlling shareholder receives a gain, due to the rescue decision. In this case, a conflict of interests will exist. For this to occur, X_2

must be sufficiently high so as to compensate that part due to the loss for minority shareholders. Alternatively, both types of shareholders may suffer a value gain at the same time when $\{f \cdot V_d - cash + X_1\} > 0$. In this case the decision will also be beneficial to minority shareholders for any $X_2 > 0$, and no horizontal agency conflict of interests will arise.

In summary bailing out the subsidiary may not be against the interests of minority shareholders in the holding company. For the bailout decision to be beneficial to all shareholders in the HC, it suffices that total common external benefits X_1 are greater than the net cost of rescue at the HC level.

However, all else being equal, the controlling shareholder's decision to bail out the distressed subsidiary will more likely harm the minority shareholders at the Holding Company. When all else is equal, and the stake of the holding company in the subsidiary, f , is relatively small (close to 0.5), the benefit or damage caused to minority shareholders can be estimated from below directly by the change in the holding premium or discount level.

B2 Internal benefits: Private benefits of control

Based on the model in section A, we now turn to the existence of private benefits of control, characterized as wealth transfers from the subsidiary's shareholders to the controlling shareholder of the holding company. This activity is customarily referred to as tunneling and it is described elsewhere in the literature, including Friedman et al. 2003. A comprehensive description of different types of tunneling activities can be found in Atanasov et al. 2008.

In our model, a total of T resources are extracted directly from the subsidiary, lowering its security value. Analogous to the previous cases, we assume that T is decomposed into two parts: T_1 which is extracted through the holding company, thereby increasing its security value, thus benefiting all its shareholders, and T_2 which is extracted directly from the subsidiary to the controlling shareholder's pockets. This is not reflected in the HC's security value, as it is received only by its controlling

shareholder. T_1 and T_2 take positive values and are not state dependent. Hence, the subsidiary's asset values are $V_u - T_1 - T_2$ and $V_d - T_1 - T_2$ in states "u" and "d" respectively.

We assume that if the financially distressed subsidiary went bankrupt, then the private benefits of control T would be lost. We also assume that cash held at the holding company is just enough to pay debt holders to avoid the subsidiary's bankruptcy, and that $V_u - T_1 - T_2 > D > V_d - T_1 - T_2 > 0$. Similarly to Box B1, Box B2 summarizes wealth values, which include security values as well as private benefits, accrued by both types of shareholders and for both states of nature in $t=1$.

Box B2

Holding Company shareholder net wealth in $t=1$

Case 2: Private Benefits of Control

		State of nature "u"	State of nature "d"
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D) + Cash) + T_2$	$h \cdot (Cash)$
	Minority	$(1 - h) \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D) + Cash)$	$(1 - h) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D) + Cash) + T_2$	$h \cdot (T_1 + f \cdot (V_d - T_1 - T_2)) + T_2$
	Minority	$(1 - h) \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D) + Cash)$	$(1 - h) \cdot (T_1 + f \cdot (V_d - T_1 - T_2))$

Bailout decision

As shown in Box B2, when state of nature is "u" internal benefits of control will remain in place. Alternatively, if state of nature is "d" and the controlling shareholder lets the subsidiary go

bankrupt, all shareholders will lose their share of the internal benefits and keep their share of the cash. On the contrary, if the controlling shareholder of the HC decides to bail out the distressed subsidiary, shareholders will keep their share of the internal benefits by paying the face value of debt using the retained earnings of the holding company.

Since, by assumption, the controlling shareholder at HC has enough voting power to unilaterally decide the subsidiary's fate, the decision will be based on the relative wealth of the controlling shareholder under the bad state of nature. Accordingly, the controlling shareholder will decide to bail out the subsidiary depending on the relative importance of his share of internal benefits and net costs of rescue. The bailout decision will be made if

$$(1-q) \cdot h \cdot \left\{ T_1 + \frac{T_2}{h} + f \cdot (V_d - T_1 - T_2) - cash \right\} > 0$$

The previous condition can be expressed in terms of the comparison of total weighted tunneling benefits for the controlling shareholder, with his net cost of rescue in the absence of tunneling, or

$$T_1(1-f) + T_2 \cdot \frac{1}{h} \cdot (1-h \cdot f) + f \cdot (V_d) - cash > 0$$

Therefore, in the presence of private benefits of control or tunneling, it is more likely that, in case of financial distress, the controlling shareholder will decide to bail out the subsidiary when the private benefits obtained by him at the holding company level are, relatively speaking, bigger (the effect of internal benefits are maximized when f is close to one and h is close to 0.5), and the level of subsidiary's debt is lower. The effect of f , the HC's stake in the subsidiary is unambiguous. This is because given T_1 , T_2 and our assumption that $V_d - T_1 - T_2 > 0$, an increase in f increases the value of the net asset rescued. Again, as in case B1, the effect of h , the controlling shareholder's stake in the Holding Company is also unambiguous. A bailout will be more likely as h decreases, because now this increases the weighted benefit received by the controlling shareholder, without any counter effect. This confirms the intuition that the lesser stake h , the more the cost of rescue will be afforded by minority shareholders.

Holding Discount

Let's first assume that the controlling shareholder decided to go bankrupt because $(1-q) \cdot h \cdot \left\{ T_1 + \frac{T_2}{h} + f \cdot (V_d - T_1 - T_2) - cash \right\} < 0$ and the securities market anticipated that. In this case the holding company would receive tunneling benefits only in the good state and would never use the existing cash; Hence, the minority shareholders' share in total security value of the HC would then be $(1-h) \cdot \left\{ q \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D)) + Cash \right\}$, their share of the NAV would be $(1-h) \cdot \left\{ q \cdot f \cdot (V_u - T_1 - T_2 - D) + Cash \right\}$ and the holding company would show a premium given by

$$(3) \text{HPremium}(\text{Tunneling, going bankrupt}) = q \cdot (1-h) \cdot T_1$$

This premium is equivalent to the expected present value of private benefits of control to be extracted from the subsidiary through the holding company. Note that the benefits exclusive to the controlling shareholder T_2 affect the decision to go bankrupt, but not the holding premium.

Let's now assume that, given the parameters of the model in the case of bankruptcy, the controlling shareholder will decide to rescue the distressed subsidiary because $T_1(1-f) + T_2 \cdot \frac{1}{h} \cdot (1-h \cdot f) + f \cdot (V_d) - cash > 0$ and the securities market anticipated that. In such a situation, the shareholders of the holding company would receive tunneling benefits in both states, incurring in their share of the rescue cost in the bad state only. The minority shareholders' share in the HC's security value would be $q \cdot \left[(1-h) \cdot (T_1 + f \cdot (V_u - T_1 - T_2 - D) + Cash) \right] + (1-q) \cdot \left[(1-h) \cdot (T_1 + f \cdot (V_d - T_1 - T_2)) \right]$, their share in NAV would be, $(1-h) \cdot \left(q \cdot \left[(f \cdot (V_u - T_1 - T_2 - D)) \right] + [Cash] \right)$ and the difference would be

$$(4) HP (tunneling, bailout) = (1-h) \cdot [T_1 - (1-q) \cdot \{D - f \cdot (V_d - T_1 - T_2)\}].$$

Expression (4) shows that, in the case of tunneling, if the market anticipates that the distressed subsidiary will be rescued by the holding company, either a holding premium or a discount will be observed, depending on whether the portion of tunneling benefits common to all shareholders of the holding company is greater than the expected net cost of rescue****. The premium or discount observed will be greater the smaller the expected cost of rescue is, which, in turn, is smaller the greater the probability of a “good” state, the smaller the holding company share in the subsidiary, and the smaller the subsidiary’s Debt is. Interestingly, it can be shown that if a bailout decision is made, the change in the level of the holding company’s premium or discount is exactly equal to the change in the minority shareholders’ wealth with respect to the bankruptcy case.

Let $\Delta(W_m^T)_{nr}^r$ be the wealth change of minority shareholders, and ΔHD_{nr}^r be the change in the holding discount level, from bankrupt or no rescue to bailout, it can be shown that $\Delta(W_m^T)_{nr}^r = \Delta HD_{nr}^r$. This means that the change in the holding discount due to an unanticipated bailout is an exact estimation of the true wealth change to minority shareholders.

The existence of private benefits of control, as well as externalities, may induce the controlling shareholder of a holding company to bailout a financially distressed subsidiaries, depending on the relative sizes of benefits and costs of rescue. When the perceived private benefits are low in comparison to the rescue costs, the controlling shareholder will let the subsidiary fail in the bad states of nature. In that scenario, the holding company’s shares will be traded at a premium with respect to the NAV, which is proportional to the expected common private benefits accrued by minority shareholders. However, when private benefits are large enough, the controlling shareholder will rescue distressed subsidiaries in order to maintain the tunneling of such benefits. Minority shareholders will trade their shares considering the expected impact of the bailout costs. Intuitively, when these costs are large compared to their share of the private benefits extracted from

**** See footnote 13

the subsidiary, the minority shareholders shares will trade at a discounted value with respect to the NAV. Notwithstanding, holding premium can be observed when the amount of private benefits is large enough.

The effects on shareholders wealth, and in the holding premium or discount will also depend on whether the bailout decision has been anticipated by the market or not in $t=0$. If the market has anticipated a bailout decision, the holding company's security value is discounted in $t=0$, the minority shareholders initial stocks are purchased and on-going trades are made at the discounted prices, and no loss of wealth occurs. Notwithstanding the above, if the subsidiary is rescued in $t=1$, the minority shareholders may eventually oppose that action. This will be treated in detail when we analyze conflicts of interest.

Now, if the decision to rescue the distressed subsidiary is not internalized by the market in $t=0$ security values, an unexpected bailout will produce an effect on the minority shareholders wealth, which can be a gain or a loss depending on the parameters of the model. The wealth effect to minority shareholders will exactly match the change in the holding premium or discount level, produced by the announcement of the decision.

Conflicts of Interest

Assume that bailout is optimal for the controlling shareholder of the holding company. This means that compared to a situation of non-rescue or going bankrupt, the controlling shareholder will suffer

a wealth increase of $(1-q) \cdot h \cdot \left\{ (T_1 + f \cdot (V_d - T_1 - T_2)) - cash + \frac{T_2}{h} \right\} > 0$. The first three terms of the

left hand side of this expression correspond to the value change accrued by the minority shareholders. Hence, the minority shareholders may eventually suffer a loss because

$(1-q) \cdot (1-h) \cdot \left\{ (T_1 + f \cdot (V_d - T_1 - T_2)) - cash \right\} < 0$, while the controlling shareholder benefits from

a gain due to the rescue decision, in which case a conflict of interests will exist. For this to occur, a T_2 sufficiently high so as to compensate the part due to the loss to minority shareholders will be

necessary. Alternatively, both types of shareholders may benefit from a wealth increase at the

same time, because $(T_1 + f \cdot (V_d - T_1 - T_2)) - cash > 0$. In this case the decision will also be beneficial to minority shareholders for any $T_2 > 0$ and no horizontal agency conflict of interest will arise. In other words, if the minority shareholders benefited from the rescue, the controlling shareholder would also benefit. Unfortunately, the reverse is not necessarily true, giving rise to potential conflicts of interest.

For the bailout decision to be beneficial for all shareholders in the holding company, it suffices that the total common external benefits T_1 are greater than the net cost of rescue at the holding company level.

It is interesting to note that the controlling shareholder's decision to bailout the distressed subsidiary will more likely harm the minority shareholders at the Holding Company, when all else is equal and the stake of the controlling shareholder in the holding company is relatively small (h is close to 0.5), producing incentives to eventually keep the smallest stake possible in order to share the rescue costs if a rescue is going to be carried out. Also note that changes in wealth caused for the minority shareholders by the bailout can be measured directly by the change in the level of the holding premium/discount.

B3 Synergies

Finally, we allow for the presence of synergies, Y . Synergies may arise from many sources vastly treated elsewhere in economics and management literature, as economies of scope, shared resources, shared markets, shared costs and distribution channels, network effects, economies of scale in joint operations, and financial synergies. In diversification and conglomeration literature, unexploited synergies are always present as a source of incentives to diversify or to conglomerate. We divide Y in Y_1 which are common synergies, which benefit all shareholders in the HC, and Y_2 , synergies that benefit the controlling shareholder exclusively, so that $Y = Y_1 + Y_2$. Common synergies Y_1 , are obtained from two different sources:

Firstly, an increase of Y_1^{Aff} in the subsidiary's security value, hence increasing the holding

company's security value, and a direct increase of Y_1^{other} in the holding company's security value coming from other subsidiaries, hence, $Y_1 = Y_1^{Aff} + Y_1^{other}$, and is shared by all shareholders in the holding company. Y_1 and Y_1^{other} are private values in the sense that they cannot be disentangled from the other asset values by the market.

Secondly, and exclusive to the controlling shareholder, we have included Y_2 , the synergy that only affects security values of companies outside the boundaries of the group, having business relations with it and owned by the controlling shareholder of the holding company..

For the existence of any kind synergies that the subsidiary belongs to the group is essential¹³. If the subsidiary didn't belong to the group, because it went bankrupt, all synergies would be lost. As in cases B1 and B2, cash held at the holding company is assumed to be the amount that must be paid to debt holders to avoid the subsidiary's bankruptcy, D . We also assume $V_u + Y_1^{Aff} > D > V_d + Y_1^{Aff}$, and $Y_1^{Aff}, Y_1^{other}, Y_2 \geq 0$ and they are not state dependent.

Box 3 summarizes wealth values, that include security and synergy values, for both types of shareholders and for both states of nature in $t=1$.

Box B3

Holding Company shareholder net wealth in $t=1$

Case 3: Synergies

		State of nature "u"	State of nature "d"
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other} + Cash) + Y_2$	$h \cdot (Cash)$
	Minority	$(1 - h) \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other} + Cash)$	$(1 - h) \cdot Cash$
HC bails	Controlling	$h \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other} + Cash) + Y_2$	$h \cdot (f \cdot (V_d + Y_1^{Aff}) + Y_1^{other}) + Y_2$

out distressed subsidiary	Minority	$(1-h) \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other} + Cash)$	$(1-h) \cdot (f \cdot (V_d + Y_1^{Aff}) + Y_1^{other})$
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Bailout decision

As shown in Box B3, when the state of nature is “u” synergies will remain in place. Now, if the state of nature is “d” and the controlling shareholder lets the subsidiary go bankrupt, all shareholders will lose their share of the synergies and keep their share of the cash. Alternatively, if the controlling shareholder decides to bail out the distressed subsidiary, shareholders will keep their share of the synergies by paying the face value of debt using the retained earnings of the holding company.

The controlling shareholder at the holding company has enough voting power to unilaterally decide the subsidiary’s fate, hence the decision will be based on his relative wealth under the bad state of nature. Accordingly, the controlling shareholder will decide to bail out the subsidiary depending on the relative importance of his share of internal benefits and his share of the net costs of rescue. The

bailout decision will be made if $(1-q) \cdot h \cdot \left\{ Y_1^{other} + \frac{Y_2}{h} + f \cdot (V_d + Y_1^{Aff}) - cash \right\} > 0$

In the presence of synergies, it is more likely that the controlling shareholder will decide to bail the subsidiary in the case of financial distress, when the synergies obtained by him at the holding company level are, relatively speaking, bigger (their effect is maximized when f is close to 1.0, and h is close to 0.5), and the level of subsidiary’s debt is lower. Again, as in cases B1 and B2, the effect of h , the controlling shareholder’s stake in the holding company is unambiguous. A bailout will be more likely the lesser h is, because now this increases the total benefit received by the controlling shareholder, without counter effects. This confirms again the intuition that the lesser stake h , the more of the cost of rescue will be afforded by minority shareholders.

Holding Discount

Let's first assume that the controlling shareholder decided to go bankrupt because

$$(1-q) \cdot h \cdot \left\{ Y_1^{other} + \frac{Y_2}{h} + f \cdot (V_d + Y_1^{Aff}) - cash \right\} < 0$$

and the securities market anticipated that. In this case the holding company would receive synergy benefits only in the good state and would never use the existing cash; Hence, minority shareholders share in security value of the HC would be

$$(1-h) \cdot \left\{ q \cdot \left[(f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other}) \right] + Cash \right\},$$

$$\text{their share in NAV would be } (1-h) \cdot \left\{ q \cdot \left[(f \cdot (V_u + Y_1^{Aff} - D)) \right] + Cash \right\}$$

$$(5) \text{HPremium}(\text{Synergies, going bankrupt}) = q \cdot (1-h) \cdot Y^{other}$$

This premium is equivalent to the expected present value of synergy benefits to be received only by the holding company. The rest of the synergy benefits, Y_2, Y^{aff} affect the decision to go bankrupt, but not the holding premium.

Let's now assume that the controlling shareholder decided to rescue the distressed subsidiary

$$\text{because } (1-q) \cdot h \cdot \left\{ Y_1^{other} + \frac{Y_2}{h} + f \cdot (V_d + Y_1^{Aff}) - cash \right\} > 0$$

and again, the securities market anticipated that. In such asituation, the shareholders of the holding company would receive their share in synergy benefits in both states of nature, incurring in their share of the cost of rescue in the bad state only. The minority shareholders' share in the security value of the holding company would be

$$q \cdot \left[(1-h) \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Y_1^{other} + Cash) \right] + (1-q) \cdot \left[(1-h) \cdot (f \cdot (V_d + Y_1^{Aff}) + Y_1^{other}) \right],$$

$$\text{their share of the NAV would be, } q \cdot \left[(1-h) \cdot (f \cdot (V_u + Y_1^{Aff} - D) + Cash) \right] + (1-q) \cdot \left[(1-h) \cdot Cash \right]$$

and the difference would be

$$(6) \text{HP}(\text{synergies, bailout}) = (1-h) \cdot \left\{ Y^{other} + (1-q) \cdot (f \cdot (V_d + Y_1^{Aff}) - D) \right\}$$

Expression (6) shows that in the presence of synergies, if the market anticipates that the distressed subsidiary will be rescued by the holding company, we could observe either a holding premium or a discount. This will depend on whether the synergy benefits received by the shareholders of the holding company, from the rest of the group, not from the subsidiary nor from outside the group, are greater than the expected net cost of rescue^{††††}. The premium or discount observed will be greater or smaller, respectively, the smaller the expected cost of rescue is, which in turn is smaller the greater the probability of a “good” state” of nature is. The premium or discount observed will be greater or smaller respectively, the greater the holding company’s share in the subsidiary is, and the smaller subsidiary’s Debt is. Interestingly, as in B2, it can be shown that if a bailout decision is made, the change in the holding company’s premium or discount is exactly equal to the change in the minority shareholders’ wealth with respect to the bankruptcy case.

As in the cases of external benefits and private benefits of control, the existence of synergies may induce the controlling shareholder of a holding company to bail out financially distressed subsidiaries, depending on the relative sizes of his benefits and his costs of rescue. When the perceived synergy benefits are low in comparison to the rescue costs, the controlling shareholder will let the subsidiary fail in the bad states of nature. In that scenario, the holding company shares will be traded at a premium with respect to the NAV. However, when synergy benefits are large enough, the controlling shareholder will rescue distressed subsidiaries in order to maintain such benefits. Minority shareholders will trade their shares considering the expected impact of the bailout costs. Intuitively when these costs are large compared to their share of the synergies obtained, the minority shareholders’ shares will trade at a discounted value with respect to the NAV. Notwithstanding, holding premium can be observed when the amount of synergies is large enough.

The effects on shareholders’ wealth, and in the holding premium or discount will also depend on whether the bailout decision has been anticipated by the market or not in $t=0$. If the market has anticipated a bailout decision, the holding company’s security value is already discounted in $t=0$,

^{††††} See footnote XXXX

the minority shareholders initial stock are purchased and on-going trades are made at the discounted prices, and no loss of wealth occurs. Notwithstanding the above, when the time comes to rescue the subsidiary in $t=1$, minority shareholders may eventually oppose that action, analogous to what has been analyzed in cases B1 and B2. This will be treated below. Now, if the decision to rescue the distressed subsidiary were not internalized by the market in $t=0$ security values, an unexpected bailout would produce an unexpected effect on the minority shareholders' wealth, which can be a gain or a loss depending on the parameters of the model. This wealth effect is equal to the change in holding premium or discount level produced by the revelation of the decision.

Conflicts of Interest

Assume that bailout is optimal for the controlling shareholder of the holding company. This means that compared to non rescue or going bankrupt, the controlling shareholder will receive a wealth gain of $(1-q) \cdot h \cdot \left\{ f \cdot (V_d + Y_1^{Aff}) - cash + Y_1^{other} + \frac{Y_2}{h} \right\} > 0$. The first three terms on the left hand side of this expression correspond to the value change accrued by the minority shareholders. Hence, minority shareholders may eventually suffer a loss $(1-q) \cdot (1-h) \cdot \left\{ \left(f \cdot (V_d + Y_1^{Aff}) - cash + Y_1^{other} \right) \right\} < 0$, while the controlling shareholder, at the same time, receives a gain due to the rescue decision, in which case a conflict of interest will exist. For this to occur, a Y_2 sufficiently high so as to compensate the part due to the loss for minority shareholders must exist. Alternatively, both types of shareholders may experiment a wealth gain at the same time, because $(1-q) \cdot (1-h) \cdot \left\{ \left(f \cdot (V_d + Y_1^{Aff}) - cash + Y_1^{other} \right) \right\} > 0$. In this case the decision will also be beneficial to minority shareholders for any $Y_2 > 0$ and no horizontal agency conflict of interest will arise. In other words, if minority shareholders benefited from the rescue, the controlling shareholder would also benefit. Unfortunately, the reverse is not necessarily true, giving rise to potential conflicts of interest. Again, this same conclusion has been reached in cases B1 and B2.

The controlling shareholder's decision to bailout the distressed subsidiary will more likely harm the

minority shareholders at the holding company, all else being equal, when the stake of the holding company in the subsidiary is relatively high (f is close to 1.0) and the stake of the controlling shareholder in the holding company is low (h is close to 0.5), which produces incentives to eventually keep the smallest stake possible in order to share the rescue costs if a rescue is going to be carried out. Also note that the wealth change caused to minority shareholders by the bailout can be measured directly by the change in holding premium or discount level.

Conclusions

We have developed a simple two period model of a pyramidal group, in an economic setting without transaction costs, information asymmetries or taxes. Our model complements other models existing in the recent literature (Almeida & Wolfenson 2006), and in the points of coincidences, reaches the same conclusions.

The main contribution of this paper has been to extend the analysis of the existence of pyramidal groups, by explicitly introducing the possibility of bailing out distressed subsidiaries of the groups, a phenomenon which is observed in the market. This has shown even more light on some of the main determinants of holding discount levels in pyramidal groups observed in the market place, and of the determinants of the time evolution and finally sudden shocks eventually observed. Additionally we have shown that when controlling shareholders decide to bail out a subsidiary, it is perfectly possible that, against conventional wisdom, the minority shareholders benefit from that decision.

We have shown that in a simple pyramidal group, rational behavior of the controlling shareholder in the case of financial distress of the subsidiary, unambiguously implies the subsidiary going bankrupt, together with a holding discount.

Nevertheless, when we allow for the existence of certain types of benefits that are commonly referred to in the securities markets, in the corporate sector and in the literature, the structure of incentives changes, and the decision to let the distressed subsidiary go bankrupt may not be rational, and holding premiums may be observed, or, considering other determinants of the holding discount, a decrease in their level would be observed.

In our model, we take as given and separately, external benefits, private benefits of control like tunneling, and synergy benefits, but we acknowledge they could be internalized by extending the time frame one period backwards to allow for the decision whether to incorporate or not a subsidiary into the business group in the presence of such benefits. This topic is treated well enough for our purposes in Almeida & Wolfenson 2006 and related literature.

Empirical and anecdotal evidence shows that a holding discount together with the bailout of subsidiaries are pervasive in Chilean pyramidal groups; arguments such as the presence of tunneling are made to justify empirical observation of holding discounts. Additionally reputation arguments are sometimes given to justify the bailout.

Our model predicts that in the presence of external benefits, the latter statement is true. Nevertheless, our model also predicts that in a pyramidal group with presence of tunneling or synergies, and financial distress of a subsidiary, a rational controlling shareholder should not bail out the subsidiary under any circumstances. If he did, a holding premium could be observed, contrary to the first argument.

Finally, our model shows that if security markets correctly anticipated the bailout or bankruptcy of a subsidiary, the premiums or discount levels are such that no gains or losses from stock trade should occur. Only in the case that an unexpected bailout is announced, an unexpected wealth change will affect minority shareholders, a change that could be measured by the sudden change in the holding discount or premium that will follow the announcement.

The widely observed holding discount together with a bailout can be inferred from our model only if external benefits are present and net bailout costs dominate synergy or tunneling benefits.

It can also be derived from our model that, against conventional wisdom, not in every case in which the controlling shareholder of the holding company decides to bail out a subsidiary in his own benefit, minority shareholders will disagree with such decision.

A summary of the main results of the model is presented in Box B4

Box B4
Holding Discount: Summary of Results

Holding Premium / Discount Level	Base Case (no benefits)	Case B1 (external benefits)	Case B2 (tunneling)	Case B3 (synergies)
Market anticipates subsidiary bankruptcy	0	0	$q \cdot (1-h) \cdot T_1$ <i>premium</i>	$q \cdot (1-h) \cdot Y_1^{other}$ <i>premium</i>
Market anticipates subsidiary is rescued	$(1-q) \cdot (1-h) \cdot (f \cdot V_d - D)$ <i>discount</i>	$(1-q) \cdot (1-h) \cdot (f \cdot V_d - D)$ <i>discount</i>	$(1-h) \cdot [T_1 - (1-q) \cdot \{D - f \cdot (V_d - T_1 - T_2)\}]$ <i>premium or discount</i>	$(1-h) \cdot \{Y^{other} + (1-q) \cdot (f \cdot (V_d + Y_1^{Aff}) - D)\}$ <i>premium or discount</i>
Unanticipated bailout				
Minority Shareholders Wealth change,	$(1-q) \cdot (1-h) \cdot (f \cdot V_d - D)$	$(1-q) \cdot (1-h) \cdot \{(f \cdot V_d - D) + X_1\}$	$(1-h) \cdot (1-q) \cdot [T_1 - \{D - f \cdot (V_d - T_1 - T_2)\}]$	$(1-h) \cdot (1-q) \cdot \{Y^{other} + (f \cdot (V_d + Y_1^{Aff}) - D)\}$
Bailout Decision				
Controlling shareholder Incentives	No	$f \cdot V_d - cash + X_1 + \frac{X_2}{h} > 0$	$(f \cdot (V_d - T_1 - T_2)) - cash + T_1 + \frac{T_2}{h} > 0$	$f \cdot (V_d + Y_1^{Aff}) - cash + Y_1^{other} + \frac{Y_2}{h} > 0$
Conflicts of Interest	No	$f \cdot V_d - cash + X_1 < 0$	$f \cdot (V_d - T_1 - T_2) - cash + T_1 < 0$	$f \cdot (V_d + Y_1^{Aff}) - cash + Y_1^{other} < 0$

Analysis of polar cases

Assume first that the only benefits across models are accrued by the controlling shareholder, this is

$X_1 = T_1 = Y_1^{aff} = Y_1^{other} = 0$. A quick inspection of Box B4 shows us that:

- a) If the market anticipates that the controlling shareholder will let the distressed subsidiary go bankrupt, the Holding discount level will be zero regardless of the type of benefit.
- b) If the market anticipates that the controlling shareholder will bailout the distressed subsidiary, this will be shown as a Holding discount in all our models. Note that in the base case there would be no economic rationale to expect a bailout of the distressed subsidiary.
- c) If the market anticipates a distressed subsidiary will go bankrupt, but an unexpected bailout is announced, the minority shareholders of the HC will suffer a wealth loss regardless of the type of benefit.

Assume now that the only benefits across models are common benefits, benefits accrued by both the controlling and minority shareholders; in this case $X_2 = T_2 = Y_2 = 0$. Box B4 shows us that:

- d) In case the market anticipates that the controlling shareholder will let the distressed subsidiary go bankrupt, the Holding will show zero discount level when no benefits or only extrenality benefits exist, and the Holding will show a premium if benefits are private benefits or synergies from other companies.
- e) In case the market anticipates that the controlling shareholder will bail out the distressed subsidiary, the Holding will show a discount level when no benefits or only extrenality benefits exist, and the Holding will show a premium or a discount if benefits are private benefits or synergies from other companies. In the last two cases, the holding will show a bigger premium or a lesser discount the greater the common benefits in relation to the corresponding cost of rescue.
- f) If the market anticipates that a distressed subsidiary will go bankrupt, but an unexpected bailout is announced, the minority shareholders of the HC will suffer a wealth loss in the no benefit or externalities cases, and a loss or a gain in the private benefits and synergy models, being the

gain in wealth greater, the greater the common benefits in relation to the corresponding cost of rescue.

Bailout Likelihood

The next box summarizes the impact of the different parameters on the likelihood of observing a bailout of a distressed subsidiary. This is the same as analyzing the impact of the different variables on the incentives of the controlling shareholder to bail out.

Box B5

Likelihood of bail out by the HC of the distressed subsidiary
Impact of the different parameters (bad state, t=1)

Effect of variable on bailout likelihood	No Benefits Base Case	External Benefits	Private Benefits	Synergies
Controlling shareholder's benefits	Null	Positive	Positive	Positive
Subsidiary Debt	Null	Negative	Negative	Negative
f : HC's stake in subsidiary	Null	Positive	positive	Positive
h : Controlling shareholder's stake in HC	Null	Negative	Negative	Negative

From Box B5 we can see that the likelihood of observing a bailout of the distressed subsidiary, whether anticipated or not, is affected in the same way regardless of the type of benefits of the model, and is bigger the greater the controlling shareholder's benefits (some of which may be shared with minority shareholders), the lesser the subsidiary's debt, the closer to 100% of the stake of the holding company in the subsidiary, and the closer to 50% of the controlling shareholder's stake in the holding company.

III Empirical Implications and Hypothesis

We have identified two main avenues to test empirical implications of our model, Panel Data and Likelihood Models, depending on the nature of the implications and on the availability of information:

On one hand, from Box 4 we can infer the following properties that can be tested empirically for the **Holding Discount Level:**

(A) If the Subsidiary is expected to go bankrupt

$$\frac{\delta HD}{\delta D} = 0; \quad \frac{\delta HD}{\delta q} \geq 0; \quad \frac{\delta HD}{\delta Benefit} \geq 0; \quad \frac{\delta HD}{\delta h} < 0 \quad \text{or}$$

(e1) $HD_{bankrupt} = HD(q^+, Benefit^+, h^-) / \text{bankruptcy is observed}$ (empirical equation 1)

(B) If the Subsidiary is expected to be rescued

$$\frac{\delta HD}{\delta D} < 0; \quad \frac{\delta HD}{\delta q} > 0; \quad \frac{\delta HD}{\delta Benefit} \geq 0; \quad \frac{\delta HD}{\delta h} < 0; \quad \frac{\delta HD}{\delta f} < 0 \quad \text{or}$$

(e2) $HD_{rescue} = HD(D^-, q^+, Benefit^+, h^-, f^-) / \text{bailout is observed}$ (empirical equation 2)

Empirical equations (e1) and (e2) can be tested using panel data analysis.

On the other hand, from Box 5 we can state that

(e3) $Likelihood\ of\ Rescue = LR(Benefits^+, Debt^-, f^+, h^-)$ (empirical equation 3)

Empirical equation (e3) can be estimated using Logit/Probit models

We can also assume that the market expectation that the subsidiary will be rescued is a monotonic function of the likelihood of rescue,

$$(e4) \quad MktExpect(Prob Rescue) = g(LR); \quad \frac{dg(LR)}{dLR} \geq 0$$

The same analysis can be used to test the propositions regarding the Holding Discount Level Changes due to a (unexpected) bailout announcement.

From Box B4 we can see that the minority shareholders's change of wealth depends on the same variables as the Holding Discount Level. In the case of Holding discount changes we can write

$$\frac{\delta\Delta HD}{\delta D} < 0; \quad \frac{\delta\Delta HD}{\delta q} > 0; \quad \frac{\delta\Delta HD}{\delta Benefit} \geq 0; \quad \frac{\delta\Delta HD}{\delta h} < 0; \quad \frac{\delta\Delta HD}{\delta f} < 0 \quad \text{or}$$

$$(e5) \quad \Delta HD_{rescue} = HD(D^-, q^+, Benefit^+, h^-, f^-) / \text{bailout is announced} \quad (\text{empirical equation 5})$$

As seen in section B1, the externalities case, the wealth change of minority shareholders is bigger than the holding discount change, precisely in a quantity proportional to the common externality benefit X_1 . So in the specific case of externalities, equation e5 must be rewritten as

$$(e5') \quad \Delta HD_{rescue} = HD(D^-, q^+, h^-, f^-) / \text{bailout is announced} \quad (\text{empirical equation 5})$$

This equation must be tested using Event Data Analysis methods.

Regarding the variables: subsidiary debt, the controlling shareholder stake in the HC and the HC's stake in the subsidiary can be extracted from financial (public) information. The probability of a bad state of nature can be approached through the business cycle of the economy. The main challenge will be to construct some proxy variables for externalities (reputation), tunneling, and synergies.

IV Venues for future research and conclusions

The results of our simple two period model allow us to interpret the presence of holding company discounts as evidence that bailing out may arise for reasons other and less conflictive than the perpetuation of tunneling benefits by the controlling shareholder.

We have shown that, against conventional wisdom, the existence of private benefits of control (tunneling) and synergies within a business group may reflect in Holding premium instead of a discount. In this way, we intend to contribute to focusing attention on factors other than tunneling to explain observed discounts.

We have also shown that the decision to bail out a subsidiary, made by the controlling shareholder of a business group, does not necessarily imply a conflict of interest between shareholders at the Holding's level, as commonly assumed. From all our results, empirical implications can be derived, and several hypothesis can be formulated for empirical analysis.

Finally, several improvements can be made in order to enrich the model; for example, extending to a multiperiod model allows us, on the one hand, to explicitly incorporate minority shareholders at the Subsidiary as active players and thus to analyze the conditions of potential conflicts of interests with them; on the other hand, it allows us to expand the set of conditions that may arise from a bailout; additionally we could analyze several information structures and their impact on our results; finally, allowing the Holding company to be levered can shine more light on the analysis of the bailout decision.

Appendix 1

This appendix describes the empirical evidence of Holding Discounts for the Chilean case. A selected set of empirical evidence has been analyzed in Lefort and González (2010) and Gálvez (2009). They report daily holding discounts for the period of January 1993 - December 2007, for nine selected business groups, which comply with a number of prerequisites: The holding company must be a public company, traded in the Chilean stock market, with no activities except for investing (being shareholder) in subsidiaries. Subsidiaries of each group, must comply with three conditions: first, their shares must be traded in the Chilean stock market, thus allowing for a true market based discount calculation; second, the book value of the subsidiaries as a whole must add up to an important percentage of the holding company's investments, thus minimizing the biases of estimating non-public company values; and third, ideally, all subsidiaries in a group must belong to the same economic sector. No constraints are imposed on the holding company regarding percentage of shares held in any Subsidiary, or being in the top tier of a more extensive business group (i.e. a holding company is allowed to be a subsidiary of another company as well).

Selected groups are: Tricahue in electric generation; Campos in the agriculture industry; Aguas Andinas in urban water & sewage; Pacífico and Oro Blanco in mining; Pampa Calichera in fertilizers; Naviera and Marinsa in maritime transportation and port agency services; and finally, Antarchile, the most diversified group in our sample, in fuel import and distribution, forestry, cellulose, fishing and mining. For these groups, subsidiaries represent 89% to 99% of the holding company's investment in companies (with the sole exception of a data point of 42%), and with the exception of the last group, all subsidiaries in the same group are very much concentrated in the same economic sector. At a monthly level, average discounts reported by Gálvez (2009), fluctuate in the 20% to 29% range, with only one group around 9%, and show a high volatility; across groups standard deviations between 0,2 and 0,8 times the average, an absolute maximum of 93%, an absolute minimum of -47% (premium). When calculations are made on a yearly basis, average holding discounts fluctuate in the 20% to 30% range, with only one group around 11%, and the absolute maximum and minimum of 60% and -7% respectively. For most of the groups, the existence of a unit root hypothesis is rejected with high frequency data, and holding discounts

show the mean reversion property in a twelve-month period on average.

Holding discount is calculated as one minus the holding company's market value of equity divided into Net Asset Value of the holding Company (NAV). NAV in turn, is calculated as the market value of the holding company's assets minus the market value of the holding company's liabilities, using face values of liabilities as proxies of their respective market value. Finally, the holding company market value of assets has two components: the market value of investment in subsidiaries, and short term assets using their face value as a proxy. Raw market data is the daily prices obtained from Economatica Data Base, financial data is released quarterly by public companies and published by the regulating entity (SVS), giving 68.880 group-day and 1.148 group-quarter observations, respectively.

Table 1 : Monthly holding company discounts for selected Chilean Business Groups
(1993 – 2009)

Holding Company	Industrial Sector	Listed subsidiaries	Holding Company Discounts in percent			Calculation period
			% of total assets	Average	Standard deviation	
Tricahue	Electric Generation	89-99	29	21.4	93/-16	1994-2009
Campos Chilenos	Agriculture Industry	90-99	20	12.5	65/-29	1993-2009
Aguas Metropolitanas	Water&Sewage Urban	95-98	21	8	36/5	2005-2009
Pacífico	Mining	43-96	9	6	26/-1	1999-2009
Oro Blanco	Mining	97-99	25	20	54/-47	2001-2009
Pampa Calichera	Fertilizers	96-99	28	12	52/-17	2000-2009
Empresas Navieras	Maritime Operations/Port	93-97	29	21	69/-27	1997-2009
Marinsa	Maritime Freight/services, Metallurgy	94-99	31	10	55/-7	1993-2009
Antarchile	Fuel distribution, Forestry, Fishing	94-99	26	5	42/18	2000-2009

Source: Lefort and González (2010) and Gálvez (2009)

Table 2 Yearly holding company discounts for selected Chilean Business Groups (1993– 2009)

Group Name	Industry	Listed subsidiaries % Explained	Total Discount %	Total Discount %	Expenses discount (S&A) %	Exp.discount (S&A+other) %	Years
			(annual average)	(annual max/min)	Annual	Annual	
Tricahue	Electric Generation	89-99	19.99	43.33/-1.09	11.15	11.07	1994-2009
Campos Chilenos	Agriculture Industry	90-99	24.80	40.79/14.94	7.78	9.71	1993-2009
Aguas Metropolitanas	Water&Sewage Urban	95-98	11.27	20.99/4.51	3.06	3.14	2005-2009
Pacífico	Mining	43-96	23.7	44.11/-7.60	9.16	10.07	1999-2009
Oro Blanco	Mining	97-99	21.20	27.68/11.83	8.59	9.71	2001-2009
Pampa Calichera	Fertilizers	96-99	30.51	38.28/18.49	8.28	15.55	2000-2009
Empresas Navieras	Maritime Operations/Port	93-97	28.74	60.23/2.86	15.81	18.51	1997-2009
Marinsa	Maritime Freight/services, Metallurgy	94-99	27.70	41.44/6.05	4.97	5.52	1993-2009
Antarchile	Fuel distribution, Forestry, Fishing	94-99	27.42	41.46/22.32	2.03	3.56	2000-2009

Source: Lefort and González (2010) and Gálvez (2009)

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